Hireracchical Decision Making Under Uncertainty In Relation To Telecommunication Industries

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Abstract: In this study, our interest was on the effect of decisions resulting from different decision makers who are independent actors and their major aim is to maximize profit in an uncertain market environment. Every firm set its decisions in the future period as a response to its competitor's policy. We employ stochastic programming and operators' model in modeling the situation. A structured model in a telecommunication environment for virtual network operators (VNO) was used without their own network facilities.

Key words: Decision making, uncertainty, stochastic programming, operators' model, virtual network operator and telecommunication.

1.0 Introduction

Decision making under uncertainty has become one of the key areas to be considered in the restructuring of some liberalized economic environment. In fact, decisions affecting the sector is no longer monopolized as the decision of the principal may directly or indirectly affects the agent(s) and vice versa. It is obvious that in the recent times, industrial sectors like telecommunications, transportation and electricity markets have been subjected to a comprehensive process of reformation which is still in progress. Modernization of the sector represents a challenge for industrial planners and academics alike. The structures of these sectors were fundamentally changed due to liberalization and the rapid pace of technological development. They were transformed from monopolies with a relatively stable and often small angled of available products to oligopolies with a broad and constantly changing variety of offered services. At the same time new actors with various different characteristics join the sector and may by means of collaboration and competition forms a variety of strategic alliances. The results of this process are for example, the absence of perfect markets, complex relationships between the decision makers and a highly dynamic and uncertain environment. The rapidly changing environment requires strong and healthy strategies and a quick adaptation to new conditions since the application of traditional microeconomic approaches will be difficult. Moreover, a decision maker must take into account the uncertainty about the environment as well as the influence of his decision on the behavior of other actors and vice versa. This suggests the analysis of such models as decision problems under uncertainty taking into account the interdependencies of several actors. The problem of decision making under uncertainty with several decision makers creates a new field of research. Depending on the viewpoint of the problem, different approaches evolved, combing concepts of decision making under uncertainty with methods of games theory or bi-level programming [1].

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Ref [2] pointed out that the concept of Stochastic programming problems with recourse allows to take into account the dynamic aspects. Ref [3] states that stochastic games can take into consideration the interplay of the decision makers as well as different types of uncertainty. Though the framework does not necessarily presuppose a hierarchical relationship between the actors. Employing sampling techniques such as, stochastic quasi-gradient methods, [4] and [5-6] give the possibility of using various representations of the uncertain variables for instance, continuous distributions. The capability of the stochastic programming framework for the modeling and analysis of strategic decision for example in telecommunication is seen in [5-6].

2.0 Definition of Terms and Theorem

2.1.8 Convex function

Let f be a real valued function defined on a non-empty convex set D in $\Re^n$,+ is said to b convex on D if,

$$f\left[\alpha x+(1-\alpha)y\right]\leq \alpha f(x)+(1-\alpha)f(y)$$

for all $x, y \in D$ and $\alpha \in [0,1]$ [7]

2.1.2 Remark 1.

If the inequality in 2.1.1 is strict, then we have Strict Convex and if the inequality is reversed, that is "$\geq$", then we have a concave function**.**

2.1.3 Random Variable

Let Ω be a sample space of a random experiment. Then, a real valued function x defined on Ω such that: $x:\Omega \rightarrow \Re$ is called a random variable.

2.1.4 Remark 2: (Classes of Random Variables)

- A random variable x is said to be discrete if its range $\Re_x$ is countable, that is the elements can be Set or listed in a sequence such as: x_1, x_2 , . . x_n.
- A random variable x is said to be continuous if its space $\Re_x$ has numbers of points equivalent to the number of points on a line segment.

2.1,5 Infinitely Divisibility of Random Variable
A random variable x and its probability distribution are said to be infinitely divisible if for each $n \geq 1$there exist independent identical distribution random variables:

$$x_1^{(n)}, x_2^{(n)}, ..., x_n^{(n)}$$

such that $x = \sum_{k=1}^{n} x_k(n) = \sum_{k=1}^{n} x_{nk}, k \geq 1$

2.1.6 Stochastic Process
Let $t \in T$ (possibly the real line $\Re$) and let $x_t(w)$ be such that $x_t : \Omega \to \Re$, then the family: $x = \{x_t(w), t \in T,\}$ is called a stochastic process. If x is such that $x_t : \Omega \to \Re^d$, then, we refer to x as a d- dimensional stochastic process [8].

2.1.7 Remark 3

In $x_t : \Omega \to \Re, w \to s, w \in \Omega, s \subset \Re$.

$x_t(w) = x(t, w)$, (ie a time continuous stochastic process) can be interpreted as the value of the process at time, t for the experiment w.

2.1.8 L^p-Process and L^p-Banded Stochastic Process
A stochastic process $x = \{x_{tw} : t \in T\}$ is an:

(i) L^p –Process if $\|x_t\|_p < \infty$

(ii) L^p –Bounded if x is an L^p –process such that $\sup_{t \in T} \|x_t\|_p < \infty$

2.1.9 Stochastic Programming
Stochastic Programming represents a framework for the analysis of decision problems characterized by uncertainty. Providing the techniques for an adequate treatment of this uncertainty, it helps to increase the accuracy and flexibility of solutions. Generally, a stochastic programming problem can be described as finding a "good" decision without knowing exactly in which state the environment will be when this decision is implemented [9].

2.2 Theorem
The Virtual Operators problem satisfies the Slater's condition:

$$Z_1\{Z / f_{2i}(z, y^*) < 0\} \neq \phi \quad ....2.2.1$$

For any feasible decision y^* of the network operator when the model parameters satisfy the conditions:

$$\Delta_1 < \frac{k_2}{c_2} \quad ....2.2.2$$

$$\Delta > \frac{k_2}{c_2} - \frac{u_2}{dc_2} \quad ....2.2.3$$

$$\Delta_1 < \Delta, 0 < u_2 \quad ....2.2.4$$

Note that this implies that customers are sensitive to a price change of the operator compared to its initial price ($c_2 \neq 0$)
Proof: When the constraints (2.2.4) are satisfied, then the domain $[\Delta_1, \Delta]X[0, u_2]$ of the VNO's problem has interior points. We show now that the complete feasible area of this problem as formed by all constraints have non empty interior for all feasible decision y^* of the Network Operator (NO), ie that the point $Z \in (\Delta_1, \Delta)X(0, u_2)$ exist which satisfy constraints which depend on the Network Operators' decisions:

$$0 \leq n_2(y^0, z) = k_2 + r_2 y_1^0 - r_{22} z_1 \quad ....2.2.5$$

$$z_2 \geq dn_2(y^0, z) = dk_2 + dr_2 y_1^0 - dr_{22} z_1 \quad ....2.2.6$$

With strict inequality, constraint (2.2.5) can be rewritten as:

$$Z_1 < \frac{k_2}{r_{22}} + \frac{r_{21} y_1^0}{r_{22}}$$

In order to be valid for all feasible Network Operator's decisions, a feasible Z_1 must also exist for the smallest possible right hand side (RHS) value of this inequality which is taken on for $y_1^0 = \Delta_1$. Then, the variable Z_1 must satisfy: $\Delta_1 < Z_1 < \frac{k_2}{r_{22}} + \frac{r_{21}}{r_{22}} \Delta_1$. This holds when the constraint (2,2,2) is satisfied.

Note that $r_{22} - r_{21} = c_2$. When $n_2(y^0, z) > o$, then, $z_2 \geq dn > 0$ holds automatically. That is only $z_2 < u_2$ for any y^0 must still be verified. For $z_2 < u_2$ constraint (2.2.6) gives

$$z_1 > \frac{k_2}{r_{22}} + \frac{r_{21}}{r_{22}} y_1^0 - \frac{u_2}{dr_{22}}$$

For any feasible y_1^0 this constraint must be satisfied by a $z_1 \in (\Delta_1 \Delta)$. the RHS of the inequality has its greatest possible value for $y_1^0 = \Delta$. In order to be

in the interior Z_1 of the feasible area for this y_1^{0}, the variable Z_1 must then satisfy
:

$$\Delta > z_1 > \frac{k_2}{r_{22}} + \frac{r_{21}}{r_{22}}\Delta - \frac{u_2}{dr_{22}}$$

which is true when constraint (2.2.3) hold. Summarizing and taking into account constraint (2.2.4), it means that under conditions (2.2.2- 2.2.4), for any feasible

$y^0 = (y_1^{\,0}, y_2^{\,0})$ a point $z = (z_1, z_2)$

exist, which satisfies all conditions of virtual operators problem with strict inequality, ie which is in the interior of the feasible area Z_1 of this problem
.

3.1 Stochastic Models of Hierarchical System

Many physical and process design problems can be described by hierarchical systems. Thus in this section, we present a stochastic formulation of the structural optimization problem. Consider the necessary and sufficient Karush-Kuln-Tucker (KKT) conditions for the lower level problem, expressed as a quadratic program to have a solution.

$cu = w$ ……………………….3.1.1a

$ku + c^T\lambda = f$ …………………3.1.1b

$w \le g$ …………………………3.1.1c

$\lambda^T(w-g) = 0$…………………3.1.1d

$\lambda \ge 0$ …………………………3.1.1e

Now suppose that F is subject to random perturbation, so that $F = F_w$. The effect of changes in F_w can be rather dramatic for certain design x and the choice of F_w the problem

$$\min_{(x,y)\in z} f(x,y) \quad \text{……………………….3.1.2a}$$

Subject to $y \in \arg \min_{x\in y} \pi(x,y)$ …………………..3.1.2b

That is (3.1.2b) is unsolvable [10]. That means in particular, that if the value of x was chosen based on some fixed vector F_w of external forces, then when the actual value of F is realized, the structure may collapse under the load. We therefore need to account for the variability of the vector F_w through a stochastic programming formulation of (3.1.2a).

$\min_{(x,y)} f(x,y) + \phi(x)$ ……………………………………………………….3.1.3a

Subject to $(x,y) \in z$ …………………………………………………….3.1.3b

$\phi(x)$ is as before the expected value of the linear recourse function,

$\phi(x) = E_w\phi(x,w)$ ……………………………………………………..3.1.4

The linear recourse problem $\phi(x,w)$ is given by

$\phi(x,w) = \min_{d\in D} q_w^{\,T} d$ …………………………………………………….3.1.5a

Subject to $Wd = k(x)y + c^T\lambda - F_w$ …………………………………….3.1.5b

Where

$D = \{(y,\lambda)\ |c_y \le g, \lambda \ge 0, \lambda^T(c_y - g) = 0\}$ ………………………… 3.1.5c

That is, the feasible D ensures that signorim contact conditions are satisfied.

4.0 models

4.1 Operator Model

In this model, we illustrate the profit of the network operators as depend on the decision of the customers and the competitors. The revenue and costs of network owners are defined thus:

Revenue: It is composed from two components.

- Revenue from service provision to customers. Given that the price charged by the network operator for its service is $q + y_1$ and its number of customers is given by:

$n_1(y,z) = k_1 - r_{11}y_1 + r_{12}z_1$. This part of revenue is $(q + y_1)n_1(y, z(y))$. This part of the revenue is: $(q + y_1)n_1(y, z(y))$

- Revenue from leasing capacity to the virtual network operator y_2z_2.

Cost:: They are costs of service provision $g_1 + e_1n_1(y_1, z(y))$

Where g_1 is the network owner fixed and e_1, the variable cost of service provision per customer. If the virtual network operator decision takes on the analytical expressions:

$$z_1(y) = \frac{k_2 - (q - e_2)r_{22} + r_{21}y_1 + dr_{22}y_2}{2r_{22}}$$ and

$$z_2(y) = \frac{d}{2}(k_2 + r_{21}y_1 + (g - e_2)r_{22} - dr_{22}y_2),$$

which implies that the decisions of VNO depend linearly on the decisions of the NO. Thus, assuming profit maximization, the network operator's decisions are the solution of the following optimization problem: Find y_1 and y_2 which maximizes

$$-(r_{11} - \frac{r_{12}r_{21}}{2r_{22}})y_1^{\ 2} + dr_{12}y_1y_2 - \frac{d^2}{2}r_{22}y_2^{\ 2} + a_1y_1 + a_2y_2$$

Subject to constraints003A

$$n_1(y, z(y)) \geq 0$$
$$\Delta_1 \leq y_1 \leq \Delta$$
$$0 \leq y_2 \leq u_1$$

Where a_1 and a_2 are expressed through parameters introduced before, Δ_1, Δ are lower and upper limits for the service price charge and u_1 is an upper bound for the price charged for leased capacity fixed by the regulation authorities.

4.2. Two- Period Model with Uncertainty and Investment in Infrastructure

Here, we want to develop a specific decision model for our environment with a Network Operator (NO) and Virtual Network Operator(s) (VNO).

Decision of the Network Owner:

y_1 =(y_{11}, y_{12}, y_{13}) – decision during period 1

y_{11} –price to charge for its service to customers during period 1

y_{12} –price to charge for capacity to the VNO during period 1.

y_{13} –maximal amount of capacity to lease to the VNO during period 1

y_2 = (y_{21}, y_{22}, y_{23}, y_{24}, y_{25}) –decision during period 2.

y_{21} –price to charge for its service to customers during period 2.

y_{22} –price to charge for capacity to the VNO during period 2.

y_{23} –maximal amount of capacity to lease to the VNO during period 2, we assume that $y_{23} \geq y_{13}$

y_{24} –amount of capacity to add at the beginning of period 2.

y_{25} –binary variable which equals 1 if the decision to all capacity to all capacity is taken and 0, if otherwise.

Decision of the Virtual Operator:

Zt = (zt_1, zt_2) –decision during period t, t= 1,2.
zt_1 –price to charge for its service to customers.
zt_2 –amount of capacity to lease from the NO.
The uncertain parameters w_t, t=1,2 determine the customer and competition model.

$$w_t = (r_1^{\ t}, r_2^{\ t} y^t)$$
$$r_{11}^{\ t} = (r_{11}^{\ t}, \ldots . r_{15}^{\ t})$$
$$r_2^{\ t} = (r_{21}^{\ t}, \ldots ., r_{23}^{\ t})$$
$$t = 1,2.$$

Where $r_1^{\ t}$ and $r_2^{\ t}$ describe the uncertainty related to the operator and the competition model respectively. Here, $r_{11}^{\ t}, r_{12}^{\ t}, r_{21}^{\ t}, r_{22}^{\ t}$ are taken from the relations:

$$n_1^{\ t} = k_1^{\ t} - r_{11}^{\ t} y_{t1} + r_{12}^{\ t} z_{t1}$$
$$n_2^{\ t} = k_2^{\ t} + r_{21}^{\ t} y_{t1} - r_{22}^{\ t} z_{t1}$$

Describing the customers' model in the case of two periods and $r_{21}^{\ t} = r_{12}^{\ t}$. Observe that the parameter $k_1^{\ t}, k_2^{\ t}$ include the number of customers and the service price decision of the respective provider in the previous time period. The parameters $e_1^{\ t}, e_2^{\ t}$ denote the variable costs of service provision per customer taken from the providers' profit expression:

$$g_1^{\ t} + e_1^{\ t} n_1^{\ t}(y, z(y))$$
$$g_2^{\ t} + e_2^{\ t} n_2^{\ t}(y, z(y))$$

5.0 Illustration

In this section, application of agent problem in Telecommunication using the model in section 4.0 and theorem in section 2.7 will be examine. For this work, we consider a Telecommunication Company –Multi-Net System Nigeria Limited situated at 406, Ikwere road, Port Harcourt (which we shall regard as Virtual Network Operator, VNO). The company buy access from all Network Operators (NO) like MTN, GLO, Starcomms, etc and in turn sells to sub- dealers and general consumers of the services. The NO supply her raw pins and other accesses which the company for instance uses to print logical (diverse recharge cards). In particular, we shall see the effect of uncertainties on the decisions of the VNO.

5.1 Decision of the Virtual Network Operator (VNO)

In this case, we shall critically examine the decision of the VNO in both periods.

Period 1: Prediction

For a given y_1 obtain a prediction $z_1(y_1)$ for the decision of the Virtual Network Operator during period 1 by solving the following problem. Find z_1 and $v_1 = (v_{11,} v_{12}, v_{13}, ..., v_{1N})$ which maximize

$$F_{10}(y_1, z_1, v_1) = \sum p_{1i}(x_2^{1i}(y_1, z_1(y_1)) - v_{1i})(q - e_2^{\ 1} + z_{11}) - r_{13}^{\ 1i} v_{1i}) - (y_{12} + x_2^{\ 1})z_{12} \qquad 5.1.1$$

Subject to constraints:

$$v_{1i} \geq \tilde{n}_2^{\ 1i}(y_1, z_1(y_1)) - \frac{1}{d_2^{\ 1}} z_{12}, i = 1 : N \qquad 5.1.2$$

$$v_{1i} \geq 0, i : N \qquad 5.1.3$$

$$\Delta_1^{\ 1} \leq z_{11} \leq \Delta^1, \qquad 5.1.4$$

$$0 \leq z_{12} \leq y_{13}, \qquad 5.1.5$$

$$\tilde{n}_2^{\ 1i}(y_1, z_1(y_1)) \geq 0, i = 1 : N, \qquad 5.1.6$$

Where v_{1i} are the potential customer of the VNO which are lost during period 1 under scenario I due to lack of capacity for service provision. The term $\tilde{n}_2^{\ 1i}$ denotes the customer number of the VNO taken into account its uncertainty about the NO's decisions:

$$\tilde{n}_2^{\ 1i}(y_1, z_1(y_1)) = k_1^{\ 1i} - r_{21}^{\ 1i}(y_{11} + \eta_1^{\ 1i}) - r_{22}^{\ 1i} z_{11},$$

where the parameter $k_1^{\ 1i}$ is also dependent on the initial customer number and the initial service price of the VNO. The structure of the profit function $F_{10}(y_1, z_1, v_1)$ is very similar to the one period competition model. The new elements are the sums containing parameters $\eta_1^{\ 1i}$ which are used to model imprecise knowledge of the virtual operator about the network owner's decisions. Furthermore, the last term under the sum represents opportunity costs for not meeting demand.

Period 2: Prediction

For a given scenario i of period 1 and a given decision $y_2^{\ i}$ of the network operator during period 2 obtain a prediction $z_2^{\ i}(y^{2i})$ for the virtual network operator decision during period 2 by solving the following problem. Find $z_2^{\ i}$ and $v_{2i} = (v_{2i1,} ..... v_{2imi})$ which maximize.

$$F_{20}(y_2^{\ i}, z_2, v_{2i}) = \sum_{j=1}^{mi} p_{2ij}\{\tilde{n}_2^{\ 2ij}(y_2^{\ i}, z_1(y_1), z_2^{\ i}(y_2^{\ i})) - v_{2ij}\}(q - e_2^{\ 2} + z_{21}) - r_{23}^{\ 2ij} v_{2ij}) - (y_{22}^{\ i} + \eta_2^{\ 2i}) z_{22}^{\ i} \qquad 5.1.7$$

Subject to constraints:

$$v_{2ij} \geq \tilde{n}_2^{\ 2ij}(y_2^{\ i}, z_1(y_1), z_2^{\ i}(y_2^{\ i})) - \frac{1}{d_2^{\ 2}} z_{22}^{\ i}, i = 1 : N, j = 1 : Mi, \qquad 5.1.8$$

$$v_{2ij} \geq 0, i = 1 : N, j = 1 : Mi, \qquad 5.1.9$$

$$\Delta_1^{\ 2} \leq z_{2i}^{\ i} \leq \Delta^2, i = 1 : N \qquad 5.1.10$$

$$0 \leq z_{22}{}^{i} \leq y_{23}{}^{i}, i = 1 : N, \ldots\ldots 5.1.11$$

$$\tilde{n}_2{}^{2ij}(y_2{}^{i}, z_1(y_1), z_2{}^{i}(y_2{}^{i})) \geq 0, i = 1 : N, j = 1 : Mi, \ldots\ldots 5.1.12$$

$$\tilde{n}_2{}^{2ij}(y_2{}^{i}, z_1(y_1), z_2{}^{i}(y_2{}^{i})) = k_1{}^{2ij} - r_{21}{}^{2ij}(y_{21}{}^{i} + \eta_1{}^{2ij}) - r_{22}{}^{2ij} z_{21}{}^{i}$$

Where v_{2ij} are the potential customers of the VNO lost during period 2 under scenario I of period 1 and scenario j of period 2 due to lack of capacity for service provision. The term $\tilde{n}_2{}^{2ij}$ denotes the customer number of the VNO taking into account its uncertainty about the NO's decisions. where the parameter $k_1{}^{2ij}$ also depend on the customer number and the service price of the VNO in the previous time period. The structure of the profit and the constraints (5.1.8) – (5.1.12) is very similar to the prediction model for period 1.

Remark

The prediction problems are quadratic programming problems which are easily solvable with standard software.

6.0 Conclusion

We focus on a framework for decision making under uncertainty when part of this uncertainty can be attributed to actions of another decision maker pursuing his own goals. A framework for modeling complex competition relationships and for evaluating strategic decisions in the telecommunication sector was applied.

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