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On the k-Modified Generalized Uniform Distribution

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ABSTRACT
The newly proposed k-Modified Generalized Uniform Distribution was considered. Some fundamental properties of the generalized distribution were derived. The theoretical and simulation results from the k-Modified Generalized Uniform Distribution indicated reasonable degree of concordance. This implies that the k-Modified Generalized Uniform Distribution can be successfully used to model observations with limited support.
Keywords: k-Modified generalized uniform distribution, Uniform distribution, Generalized distribution, Moments, Shape parameter, Mills ratio, Restricted support, Exponentiated distribution

INTRODUCTION

The Uniform Distribution (U(0,1)) is often used in distribution theory because its probability density function (pdf) and cumulative distribution function (cdf) are defined as f(x) =1 and F(x) = x. These probability functions are simple and very flexible and these make the distribution to be appealing to many researchers in the development of generalized distributions from parent baseline distributions. This implies that the Uniform Distribution U (0, 1) allows complex characterizations of properties of given density functions and their corresponding distribution functions. The Uniform Distribution U (0, 1) has also found extensive applications in the simulation of random variables based on the quantile function and this makes it attractive in the formulation of generalized distributions. The literature search shows the existence of the following generalized distributions: Beta-generated distributions by Eugene *et. al*. (2002) and Jones (2004); Kumaraswamy generalized (Kum-G) distribution by Cordeiro and de Castro (2011); McDonald generalized (Mc-G) distribution by Alexander *et. al*. (2012); Gamma-generated type-1 distributions by Zografos and Balakrishnan (2009) and Amini *et. al.* (2014), Gamma-generated type-2 distributions by Ristic and Balakrishnan (2012) and Amini et. *al. (2014)*, Exponentiated generalized (exp- G) distribution by Cordeiro *et. al.* (2013); Odd Weibull-generated distribution by Bourguignon *et. al*. (2014); Length-Biased Weighted Maxwell distribution by Modi (2015); Transmuted additive Weibull distribution by Elbatal and Aryal (2016); Exponentiated Weibull generated distribution by Hassan and Elgarchy (2016); Exponentiated Kumaraswamy power function distribution by Bursa and Kadilar (2017); Size-Biased Lindley distribution by Ayesha (2017); Quasi-Transmuted distribution by Osowole and Ayoola (2019); Generalized Length Biased Exponential distribution by Maxwell *et. al.* (2019); Kumaraswamy Generalized Kappa distribution by Nawaz *et. al.* (2020); Generalized family of exponentiated and transmuted distributions by Anmad (2020) and Length Biased Quasi-Transmuted Uniform distribution by Osowole and Onyeze (2020).

This study is therefore an additional attempt to expand the body of existing knowledge on generalized distributions by proposing the k-Modified Generalized Uniform Distribution (kMGUD).

The k-Modified Generalized Uniform Distribution

The Generalized Uniform (GU) probability density function (pdf) according to Lee (2000) is defined as

$$f_{GUD}(x:a,b)=\frac{a+1}{b^{a+1}}x^{a},0<x<b;-1<a \qquad \text{……………………………………(1.0)}$$

with the cumulative distribution function (cdf) defined as

$$F_{GUD}(x:a,b)=\left(\frac{x}{b}\right)^{a+1}\ 0<x<b;-1<a \qquad \text{……………………………………(2.0)}$$

where a and b are the shape and scale parameters respectively. Lee (2000) gave the mean and variance of the GU distribution as

$$E_{GUD}(X)=mean=\left(\frac{a+1}{a+2}\right)b \qquad \text{……………………………………(3.0)}$$

and

$$V_{GUD}(X)=\text{var}\,iance=\left(\frac{a+1}{(a+2)^{2}(a+3)}\right)b^{2} \qquad \text{……………………………(4.0)}$$

We note that (1.0) yields the conventional uniform distribution, U (0, 1) when a and b are chosen such that a = 0 and b = 1. The k-Modified Generalized Uniform Distribution (kMGUD) using Ali *et. al.* (2007) and Ramires *et. al.* (2019) approaches has pdf and cdf defined as

$$g_{kMGUD}(x:a,b,k)=\frac{(a+1)(k+1)\left(\frac{x}{b}\right)^{(a+1)(k+1)}}{x},0<x<b,-1<a,-1<k \qquad \text{……………………….(5.0)}$$

where k is an additional shape parameter

$$G_{kMGUD}(x:a,b,k)=\left(\frac{x}{b}\right)^{(a+1)(k+1)},0<x<b,-1<a,-1<k \qquad \text{……………………….(6.0)}$$

We note that (5.0) was obtained from the relationship below

$$g_{kMGUD}(x:a,b,k)=\left[G_{kMGUD}(x)\right]'=\frac{d}{dx}\left[G_{kMGUD}(x)\right]=\frac{d}{dx}\left(\left[F_{GUD}(x)\right]^{k+1}\right)$$

$$= \frac{d}{dx}\left(\left(\frac{x}{b}\right)^{(a+1)(k+1)}\right) \text{……………………….(7.0)}$$

We note that (5.0) yields the conventional uniform distribution, U (0, 1) when a = 0, b = 1 and k = 0. Also, (5.0) yields the GU distribution when a = 0 and b = 1. The plots of the pdf and cdf of the k-Modified Generalized Uniform Distribution are given below in Figures (1.0) and (2.0).

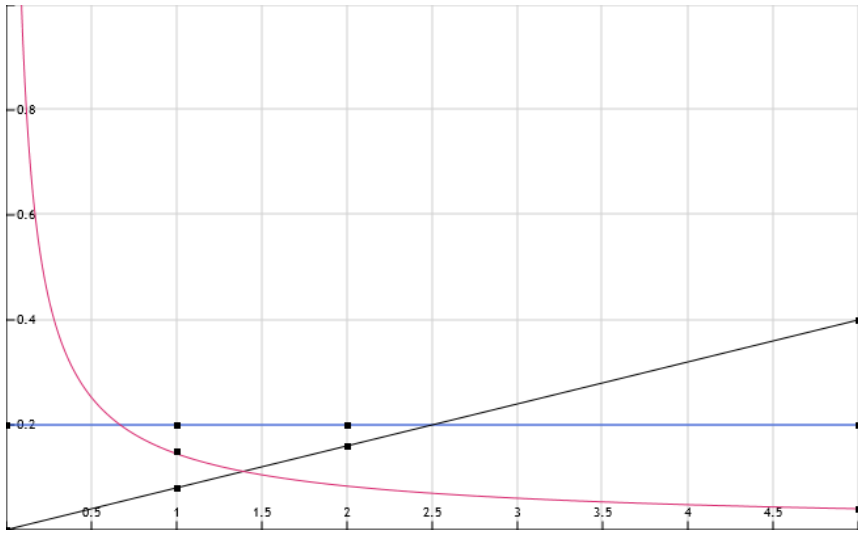

Figure 1.0: The pdf plot of the k-Modified Generalized Uniform Distribution at different values of x

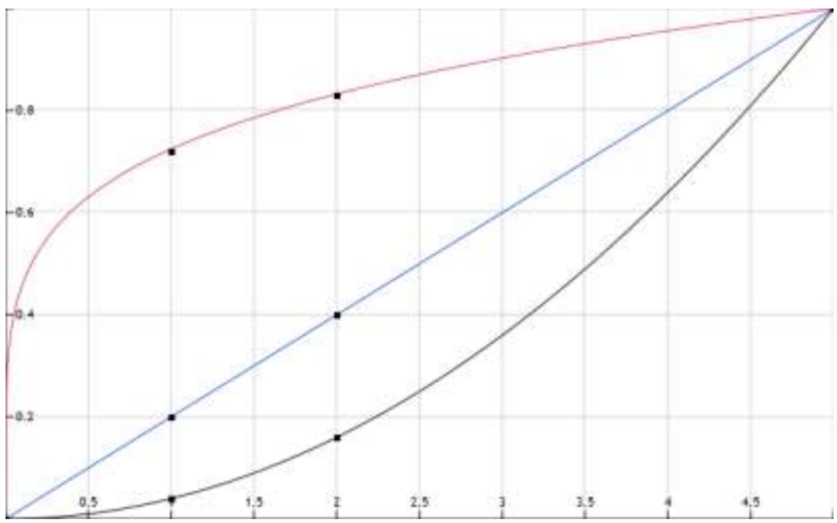

Figure 2.0: The cdf plot of the k-Modified Generalized Uniform Distribution at different values of x

The pdf and cdf plots above were plotted for predefined values of the additional shape parameter, k at a = 0 and b = 5 for different values of x.

Some Characterizations of the k-Modified Generalized Uniform Distribution
Moments

The r[th] raw moment for the k-Modified Generalized Uniform Distribution is

$$\mu_r^1 = E(X^r) = \int_0^b x^r \; g_{kMGUD}(x)\, dx$$

$$= \int_0^b x^{r-1}[(a+1)(k+1)\left(\frac{x}{b}\right)^{(a+1)(k+1)}]dx$$

$$= \frac{(a+1)(k+1)b^{r-(a+1)(k+1)+ak+k+a+1}}{r+k(a+1)+a+1} = \frac{(a+1)(k+1)b^r}{a(k+1)+r+k+1}, b>0, (a,k)>-1$$

Specifically,

$$\mu_1^1 = \mathrm{E}_{kMGUD}(X) = mean = \left[\frac{(a+1)(k+1)b}{a(k+1)+k+2}\right], (a,k)>-1, b>0$$

Also,

$$\mu_2^1 = E_{kMGUD}(X^2) = \left[\frac{(a+1)(k+1)b^2}{a(k+1)+k+3}\right], (a,k)>-1, b>0$$

From the

μ_1^1 *and* μ_2^1 above, the variance of X, $\mathrm{V}(X) = E(X-\mu)^2 = \mu_2 = \mu_2^1 - (\mu_1^1)^2$

That is, $V_{kMGUD}(X) = \left[\frac{(a+1)b^2(k+1)}{(ak+a+k+2)^2(ak+a+k+3)}\right]; (a,k)>-1, b>0$. The variance can be seen to reduce to (4.0) when k = 0 and to $\frac{1}{12}$, the variance of the conventional uniform distribution U (0, 1) when a = 0, b =1 and k = 0. Additionally,

$$\mu_3^1 = \mathrm{E}_{kMGUD}(X^3) = \left[\frac{(a+1)(k+1)b^3}{a(k+1)+k+4}\right], (a,k)>-1, b>0$$

Using the fact that

$\mu_3 = \mathrm{E}(X-\mu)^3 = \mu_3^1 - 3\mu\mu_2^1 + 2\mu^3,$ then the coefficient of skewness $\left(\alpha_3 = \frac{\mu_3}{\sigma^3}\right)$ is

$$\alpha_3 = \left(\frac{-2(a+1)b^3(k+1)(ak+a+k)}{(ak+a+k+2)^3(ak+a+k+3)(ak+a+k+4}\right) / \left(\frac{(a+1)b^2(k+1)}{(ak+a+k+2)^2(ak+a+k+3)}\right)\left(\sqrt{w}\right); b>0, (a,k)>-1$$

where $w = \sqrt{\frac{(a+1)b^2(k+1)}{(ak+a+k+2)^2(ak+a+k+3)}}$

Coefficients of Variation and Dispersion

The coefficients of variation and dispersion for the k-Modified Generalized Uniform Distribution are $CV = \frac{\sigma}{E(X)}$ *and* $CD = \frac{\sigma^2}{E(X)}$. Specifically,

$$CV = \frac{\sqrt{\frac{(a+1)b^2(k+1)}{(ak+a+k+2)^2(ak+a+k+3)}}}{\left(\frac{(a+1)(k+1)b}{a(k+1)+k+2}\right)}$$

and $$CD = \frac{(a+1)b^2(k+1)[a(k+1)+k+2]}{(ak+a+k+2)^2(ak+a+k+3)b(a+1)(k+1)}$$ for b > 0 and (a, k) > -1

Moment Generating Function

The moment generating function for the k-Modified Generalized Uniform Distribution is defined as

$$\begin{aligned} M_X(t) = \mathrm{E}(e^{tX}) &= \int_0^b e^{tX} g_{kMGUD}(x)dx \\ &= \int_0^b \left[1 + tX + \frac{(tX)^2}{2!} + \ldots\right] g_{kMGUD}(x)dx \\ &= \int_0^b \sum_{j=0}^{\infty} \frac{t^j}{j!} X^j g_{kMGUD}(x)dx \end{aligned}$$

$$= \sum_{j=0}^{\infty} \frac{t^j}{j!} \mu_j^1$$

$$= (a+1)(k+1) \sum_{j=0}^{\infty} \frac{t^j}{j!} \left[\frac{b^j}{a(k+1) + j + k + 1} \right]; b > 0,\ (a,k) > \text{-1}$$

Characteristic Function

The characteristic function for the k-Modified Generalized Uniform Distribution is defined as

$$\phi_x(t) = M_X(it)$$

$$= \mathrm{E}(e^{itX})$$

$$= \int_0^b e^{itX} g_{kMGUD}(x) dx$$

$$= (a+1)(k+1) \sum_{J=0}^{\infty} \frac{(it)^j}{j!} \left[\frac{b^j}{a(k+1) + j + k + 1} \right]; b > 0,\ (a,k) > \text{-1}$$

Cumulant Generating Function

The cumulant generating function of a random variable X from the k-Modified Generalized Uniform Distribution is defined as

$$K_X(t) = In \left[M_X(t) \right]$$

$$= In \left[\sum_{J=o}^{\infty} \frac{t^j}{j!} \mu_j^1 \right]$$

$$= In \left[(a+1)(k+1) \sum_{J=0}^{\infty} \frac{t^j}{j!} \left[\frac{b^j}{a(k+1) + j + k + 1} \right] \right]; b > 0,\ (a,k) > \text{-1}$$

Hazard Function

The hazard function for the k-Modified Generalized Uniform Distribution is defined as

$$h(x) = \frac{g_{kMGUD}(x)}{1 - G_{kMGUD}(x)}$$

$$= \frac{(a+1)(k+1) \left(\frac{x}{b} \right)^{(a+1)(k+1)} \left(\frac{1}{x} \right)}{1 - \left(\frac{x}{b} \right)^{(a+1)(k+1)}}; b > 0, (a,k) > \text{-1}$$

The plot of the hazard function is given below for some selected values of the parameters. The hazard function shows a decreasing, an increasing and a reversed bathtub pattern for varying values of the parameters.

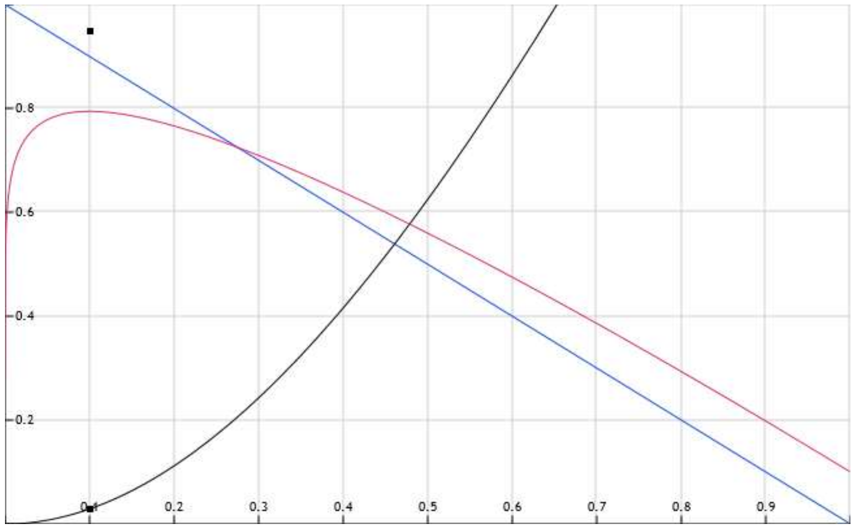

Figure 3.0: The plot of the hazard function of the k-Modified Generalized Uniform Distribution at different values of x

The plot of the hazard function was obtained for varying values of the additional shape parameter, k at a = 0 and b = 5 for different values of x.

Reverse Hazard Function

The reserve hazard function for the k-Modified Generalized Uniform Distribution is defined as

$$h_r(x) = \frac{g_{kMGUD}(x)}{G_{kMGUD}(x)}$$

$$= \frac{(a+1)(k+1)\left(\frac{x}{b}\right)^{(a+1)(k+1)}\left(\frac{1}{x}\right)}{\left(\frac{x}{b}\right)^{(a+1)(k+1)}}$$

$$= \left(\frac{(a+1)(k+1)}{x}\right); b > 0, (a,k) > -1$$

The plot of the reverse hazard function is given below in Figure (4.0)

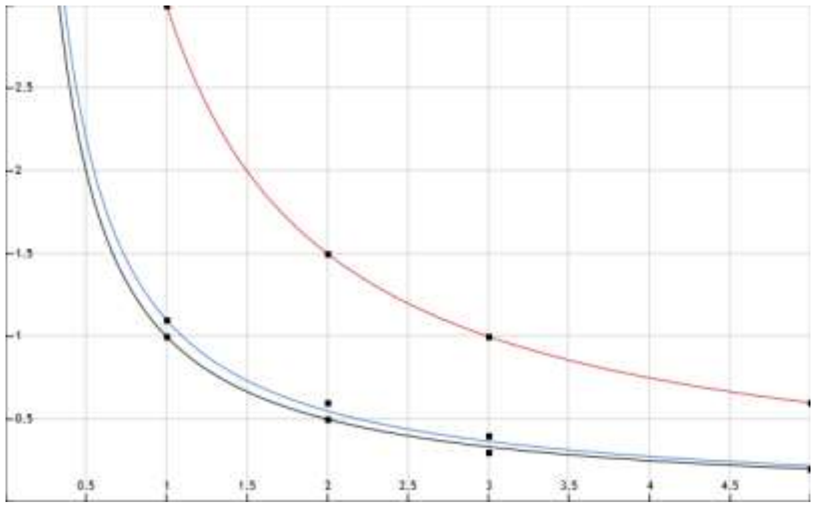

Figure 4.0: The plot of the reverse hazard function of the k-Modified Generalized Uniform Distribution at different values of x

The plot of the reserve hazard function was obtained for varying values of the additional shape parameter, k at a = 0 and b = 5 for different values of x. The function is indeed decreasing for all the values considered for the plotting.

Survival Function

The survival function for the k-Modified Generalized Uniform Distribution is defined as as

$$S_X(x) = 1 - G_{kMGUD}(x)$$

$$= 1 - \left(\frac{x}{b}\right)^{(a+1)(k+1)} \quad ; b > 0, (a,k) > -1$$

The plot of the survival function is shown below in Figure (5.0) .The plot shows the survival function of the k-Modified Generalized Uniform Distribution at a = 0 and b = 5 for varying values of the additional shape parameter.

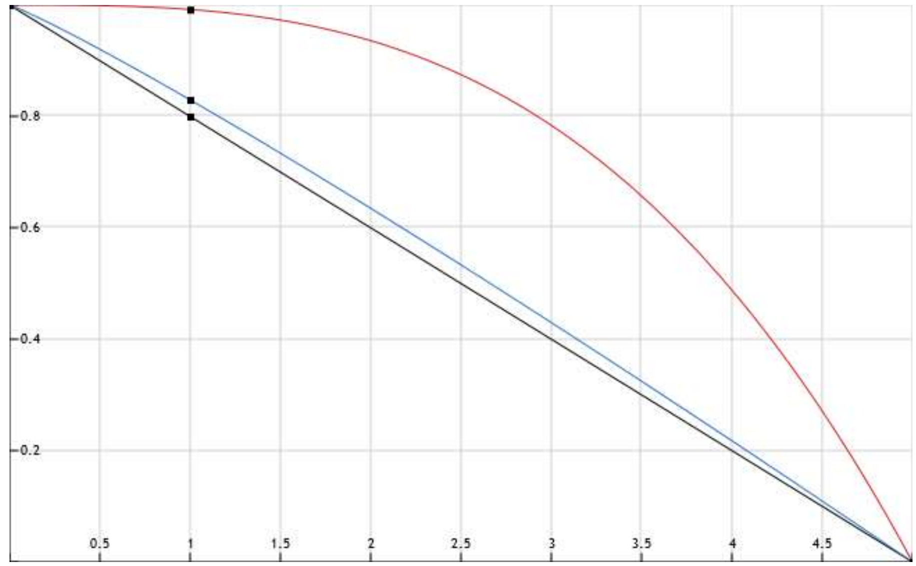

Figure 5.0: The plot of the survival function of the k-Modified Generalized Uniform Distribution at different values of x

Mills Ratio

The Mills Ratio of the k-Modified Generalized Uniform Distribution is defined as

$$\left(\frac{1}{reserve\ hazard\ function}\right)=\left(\frac{x}{(a+1)(k+1)}\right);b>0,(a,k)>-1$$

The plot of the Mills Ratio is shown below in Figure (6.0)

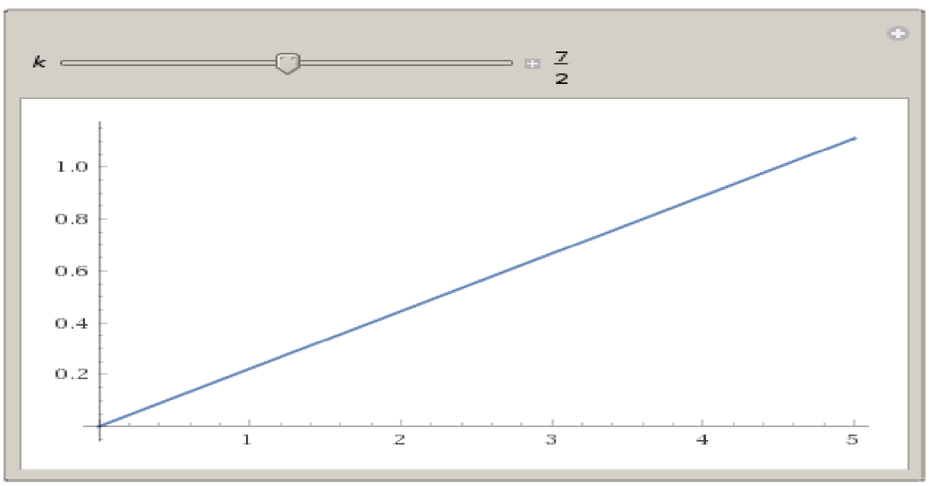

Figure 6.0: The plot of the mills ratio of the k-Modified Generalized Uniform Distribution for $0 < x < 5$

The plot of the Mills Ratio is shown above in Figure (6.0) .The plot shows that the Mills Ratio is a linear function, the function increases as x increases for the k-Modified Generalized Uniform Distribution at 0 < x < 5 and 2 < k < 5.

Odd Function

The odd function of the k-Modified Generalized Uniform Distribution is

$$ODD_X(x) = \frac{G_{kMGUD}(x)}{S_{kMGUD}(x)} = \left(\frac{\left[\frac{x}{b}\right]^{(a+1)(k+1)}}{1-\left[\frac{x}{b}\right]^{(a+1)(k+1)}} \right); b > 0, (a,k) > -1$$

The plot of the odd function is shown below in Figure (7.0)

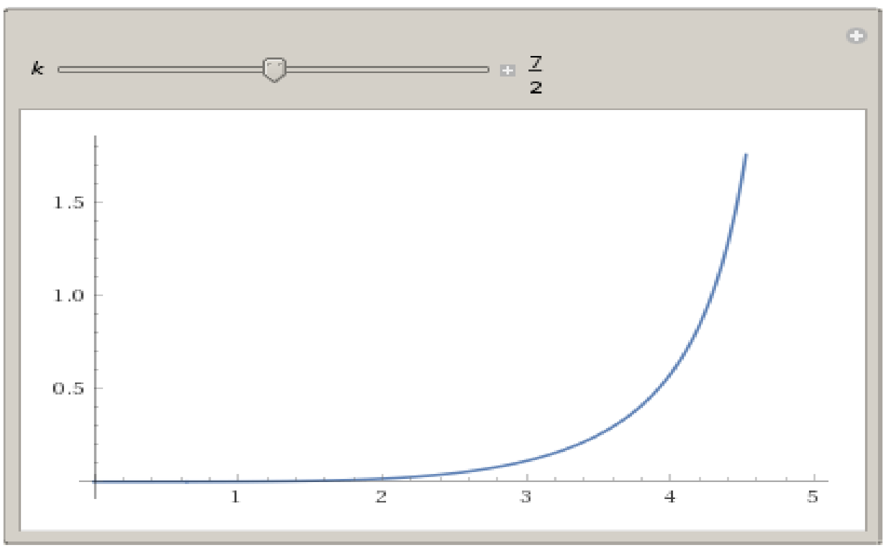

Figure 7.0: The plot of the odd function of the k-Modified Generalized Uniform Distribution for 0 < x < 5

The plot of the odd function is shown above in Figure (7.0) .The plot shows that the function is J-shaped and it increases as x increases for 0 < x < 5 and 2 < k < 5.

Cumulative Hazard Function

The cumulative hazard function for the k-Modified Generalized Uniform distribution is

$$H_{kMGUD}(x) = -Log_e\left[S_{kMGUD}(x)\right] = -Log_e\left[1-\left(\frac{x}{b}\right)^{(a+1)(k+1)}\right]; b>0, (a,k)>-1$$

The plot of the cumulative hazard function is shown below in Figure (8.0)

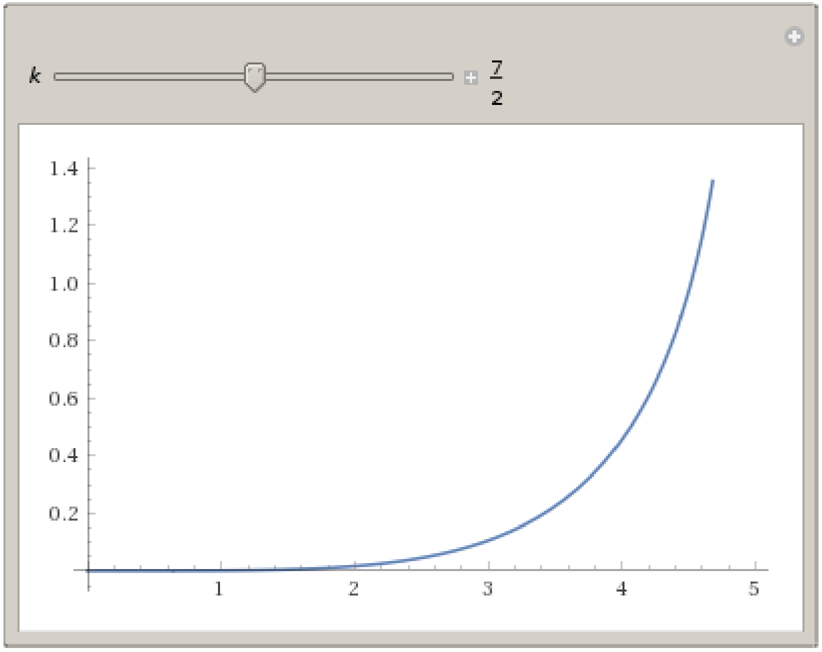

Figure 8.0: The plot of the cumulative hazard function of the k-Modified Generalized Uniform Distribution for 0 < x < 5

The plot of the cumulative hazard function is shown above in Figure (8.0) .The plot shows that the function is J-shaped and it increases as x increases for 0 < x < 5 and 2 < k < 5. This is similar to the plot of the odd function above.

Order Statistics

Let $X_{(1)}$, $X_{(2)}$,…, $X_{(n)}$ be order statistics from the random sample X_1, X_2, …., X_n from the k-Modified Generalized Uniform Distribution with $g_{kMGUD}(x)$ and $G_{kMGUD}(x)$, the r[th] order statistic where $1 \leq r \leq n$ is given as

$$h_{(r)}(x) = \frac{n!}{(r-1)!(n-r)!} g_{kMGUD}(x)\left[G_{kMGUD}(x)\right]^{r-1}\left[1-G_{kMGUD}(x)\right]^{n-r}$$

By setting r = n and r =1 in the function ($h_{(r)}(x)$) above, we have the distributions for the largest and lowest order statistics. For the largest order statistic, we have that

$$h_{(r=n)}(x) = n\left[\frac{(a+1)(k+1)\left(\frac{x}{b}\right)^{(a+1)(k+1)}}{x}\right]\left(\left[\frac{x}{b}\right]^{(a+1)(k+1)}\right)^{n-1} ;b>0,(a,k)>-1$$

For the lowest order statistic, we have that

$$h_{(r=1)}(x) = n\left[1-G_{kMGUD}(x)\right]^{n-1} g_{kMGUD}(x)$$

$$= n\left[1-\left(\frac{x}{b}\right)^{(a+1)(k+1)}\right]^{n-1}\left[\frac{(a+1)(k+1)\left(\frac{x}{b}\right)^{(a+1)(k+1)}}{x}\right];b>0,(a,k)>-1$$

Random Number Generation

Random numbers can be generated for the k-Modified Generalized Uniform Distribution using the quantile function as follows:

$$Let\ \left(\frac{x}{b}\right)^{(a+1)(k+1)} = u \qquad so\ that$$

$$In\left[\left(\frac{x}{b}\right)^{(a+1)(b+1)}\right] = u \Rightarrow (a+1)(k+1)In\left(\frac{x}{b}\right) = In\ u \Rightarrow In\left(\frac{x}{b}\right) = \frac{In\ u}{(a+1)(k+1)} \Rightarrow In\ x - In\ b = \frac{In\ u}{(a+1)(k+1)}$$

where U is a random variable from the Uniform (0,1). By further algebraic simplification,

$$x = \exp\left(\frac{In\ u}{(a+1)(k+1)} - In\ b\right)$$

Specifically, the simulation study at a = 0, b = 1 and k = 1 will be considered in this study.

RESULTS

Table 1.0: Theoretical Results from the kMGUD

Summary Statistics	Estimate
mean	0.66670 (0.7)
median	0.70710 (0.7)
variance	0.05560 (0.6)
standard deviation	0.23570 (0.2)
skewness	-0.56570 (-0.6)
coefficient of variation	0.35353 (0.4)
coefficient of dispersion	0.08340 (0.1)

Figures in bracket represent approximation to 1 decimal place

Table (1.0) above gives the estimates of some summary statistics from the k-Modified Generalized Uniform Distribution (kMGUD). The approximated estimates to 1 decimal place are also shown in brackets. These theoretical estimates were obtained for a = 0, b =1 and k = 1.

Table 2.0: Simulation Results from the kMGUD

Simulated Results			Simulated Results		
Sample Size	**Summary Statistics**	**Estimate**	**Sample Size**	**Summary Statistics**	**Estimate**
200	mean	0.69728 (0.7)	1000	mean	0.67373 (0.7)
	median	0.73969 (0.7)		median	0.71547 (0.7)
	variance	0.04679 (0.1)		variance	0.05418 (0.1)
	standard deviation	0.21631 (0.2)		standard deviation	0.23276 (0.2)
	skewness	-0.70521 (-0.7)		skewness	-0.58402 (-0.6)
	coefficient of variation	0.31022 (0.3)		coefficient of variation	0.34548 (0.4)
	coefficient of dispersion	0.06710 (0.1)		coefficient of dispersion	0.08042 (0.1)
500	mean	0.67756 (0.7)	2000	mean	0.66449 (0.7)
	median	0.69846 (0.7)		median	0.70390 (0.7)
	variance	0.05213 (0.1)		variance	0.05424 (0.1)
	standard deviation	0.22832 (0.2)		standard deviation	0.23286 (0.2)
	skewness	-0.62309 (-0.6)		skewness	-0.57806 (-0.6)
	coefficient of variation	0.33697 (0.3)		coefficient of variation	0.35043 (0.4)
	coefficient of dispersion	0.07694 (0.1)		coefficient of dispersion	0.08161 (0.1)

Figures in bracket represent approximation to 1 decimal place

The simulation results in Table (2.0) are the simulated results obtained when a = 0, b =1 and k = 1 respectively for sample sizes 200, 500, 1000 and 2000. There is a concordance between the theoretical and simulated results as shown in Tables (1.0) and (2.0) for the selected summary statistics. This implies that the theoretical characterizations of the k-Modified Generalized Uniform Distribution presented earlier in this study are valid for the proposed distribution. The results of the study are in tandem with Lee (2000) and Ramires *et. al*. (2019).

CONCLUSION

This study proposed the k-Modified Generalized Uniform Distribution and derived some of its essential properties. It further considered the theoretical and simulated results from the new distribution. The two results were seen to be in tandem with each other. The variances from the two results were consistently lower than the variance from the parent distribution. This indicates that the k-Modified Generalized Uniform Distribution is superior to the baseline Generalized Uniform Distribution. The k-Modified Generalized Uniform Distribution, as an exponentiated distribution, is therefore expected to provide a better fit for observations having restricted support.

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