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Analytical Comparison of Model Information Criteria in ASARIMA Identification: Evidence from Benin Monthly Precipitation

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ABSTRACT
The paper is concerned with optimal forecasts performance for Benin monthly rainfall (BMRF) pattern in Edo State, Nigeria using adjusted SARIMA (ASARIMA) model. It employed Amaefula forecast criterion (AFC) to compare the models selected by different information criteria such as Akaike information criterion (AIC), Schwarz Bayesian information criterion (SBIC), Hannan and Quinn(HQ), and forecast prediction error(FPE) to identify the optimal model. . The BMRF data span from 1981M1 to 2016M12. The order of integration test (OIT) adopted shows that BMRF data is integrated order zero (I(0)) in its regular series. The evidence of significant seasonal oscillation in the ACF and PACF reveal the need for seasonal differencing of MRF data. Using an iterative algorithm for calculating least squares estimates of the model parameters, 19 possible ASARIMA (P, D, Q)$_{12}$ models were compared; AIC, SBIC and HQ preferred ASARIMA(1, 1, 2)$_{12}$, ASARIMA(0, 1, 1)$_{12}$ and ASARIMA(1, 1, 5)$_{12,}$ respectively. FPE supports the choice of SBIC. The three selected models were subjected to forecast optimality test using the AFC and ASARIMA (1, 1, 2)$_{12}$ is found most appropriate. The diagnostic test indicates adequacy of the fitted model. Hence, ASARIMA (1, 1, 2)$_{12}$ model is recommended for forecasting BMRF pattern and creating inter-mediate warning against erosion and flood in Benin.
Keywords: AFC, AIC , ASARIMA model, BMRF, optimality forecast production, SBIC.
MSC: 62-XX, 62-1XX, 62-02

1. INTRODUCTION

Autoregressive integrated moving average (ARIMA) and seasonal ARIMA models have been used by many researchers over decades in the modeling and forecasting time series data such as macroeconomic and financial time series data. ARIMA and seasonal ARIMA have been extended to predicting climatic factors like rainfall, temperature and humidity across time and space. And since the introduction of ARIMA framework by Box and Jenkins in 1970, many researchers have used the model in forecasting precipitation pattern in different countries, including Nigeria. However, additional dimension to the existing SARIMA model was the study of Amaefula(2021) who introduced the adjusted seasonal ARIMA (ASARIMA) model basically on condition where a univariate time series data exhibits some seasonal non-stationary features and at the same exhibits zero unit root in the regular series. The major advantage of ASARIMA is to avoid redundancy in the non-seasonal parameters, and empirical findings showed better out-put performance than SARIMA models.

Practically, contradictory results abound in literature, different identified models for the same univariate time series have raised questions like; what is the optimal model when different information criteria are used for identification? What is the best information criteria to be used in model selection for maximum forecasts output? Attempt to answer these questions empirically in this paper is the missing link in previous researches and it is the essence of this study.

Benin is an ancient historical city in the Benin Empire, and was the capital of the defunct Bendel State, now the present capital of Edo State. It has a tropical climate with more rainfall in the summer than in the winter. The winter season is fairly cold, reaching about 27.5 ^{0}C. The month of January is mostly the driest month with about 9 mm of rainfall. The summer is hot reaching about 34 ^{0}C by day and 25 ^{0}C at night. The month of March is the hottest weather with an average of about 29 ^{0}C. The weight season begins in July and ends in October with a pick in August with about 25 ^{0}C average. The coldest period occurs in August with temperature attaining 34 0C by day and 25 ^{0}C at night. The warmest month of the year is April, with an average temperature of 27.5 °C. In July, the average temperature is 24.5 °C.

The problem of getting good drinking water in many urban and rural dwellers in the south-southern region of Nigeria has increasingly become a challenging task because vast oil spillage which have polluted the creeks, streams and rivers that serve as good sources of drinking water to many homes. However, studying monthly rainfall pattern in Benin City and generating forecasts values of the precipitation can be relevant to erosion and flood control, agriculture and plant growth, adequate environmental and water resource management and developing an adequate time series model for Benin monthly rainfall (BMRF) pattern cannot be over emphasized.

Since the introduction of ARIMA and SARIMA models over four decades by Box and Jenkins(1976), many researchers across the globe have modeled yearly rainfall pattern using ARIMA(p, d, q) framework and monthly rainfall pattern using SARIMA $(p, d, q) \times (P, D, Q)_s$ According to Yusuf and Kane(2012), rainfall is natural climatic occurrences and its prediction remains a difficult challenge as a result of climatic variability. In Tamilnadu India, SARIMA$(0, 1, 1)x(0, 1, 1)_{12}$ was found most preferable model fit for monthly rainfall (Nirmala *et. al.*, 2010).

In Malaaca and Kuantan of Malaysia, SARIMA $(1, 1, 2) \text{ x } (1, 1, 1)_{12}$ and $(4, 0, 2) \text{ x } (1, 0, 1)_{12}$ models were fitted respectively for their monthly rainfall (Yusuf and Kane, 2012). The ARIMA method was also used by Muhammet (2012) to predict the temperature and precipitation in Afyonkarahisar Province, Turkey, until the year 2025, and found an increase in temperature according to the quadratic and linear trend models.

A periodical rainfall data was modeled for Port Harcourt city, South-Southern Nigeria using SARIMA$(0, 0, 0)x(2, 1, 0)_4$ model by (Osarumwense, 2013). Modeling seasonal rainfall data was also investigated in Port Harcourt and SARIMA $(5,1,0)(0,1,1)_{12}$ was identified and established to be adequate for modeling and forecasting the amount of rainfall in the area (Etuk *et al.*, 2013). Rainfall data pattern was examined in the Ashanti region of Ghana and SARIMA $(0, 0, 0)x(2, 1, 0)_{12}$ was fitted (Abdul-Aziz *et al.*, 2013).

SARIMA model was adopted in studying monthly rainfall data for Gadaref rainfall station, Sudan. The autocorrelation structure suggests three multiplicative SARIMA models, namely: $(0, 0, 0)x(0, 1, 1)_{12}$, $(0, 0, 1)x(0, 1, 1)_{12}$ and $(0, 0, 1)x(2, 1, 1)_{12}$. The first model was deemed most appropriate for forecasting rainfall in the region (Etuk and Mohamed, 2014). Seasonal autoregressive integrated moving average (SARIMA) model was adopted by Anitha et al.,(2014) to forecast the monthly mean of the maximum surface air temperature of India. Their results showed that there is a trend in the monthly mean of maximum surface air temperature in India.

The seasonal ARIMA modeling and forecasting of rainfall in Warri Town, Nigeria for the period 2003-2012 was examined and the Seasonal ARIMA $(1, 1, 1)$ $(0, 1, 1)_{12}$ fitted was found to meet the criterion of model parsimony and model adequacy check showed that the model was appropriate, (Eni and Adeyeye, 2015). Again, using monthly data spanning from 1996 to 2011 obtained from National Root Crops Research Institute Umudike in Nigeria, SARIMA $(0, 0, 0)$ $(0, 1, 1)_{12}$ model was considered the best fitted model for forecasting monthly rainfall in Umuahia, Aba state (Akpanta *et al.,* 2015).

Time series analysis of monthly rainfall for Oshogbo Osun State, Nigeria was studied using data from 2004 to 2015. The time plot reveals that rainfall data showed a high level of volatility characterized by seasonal and irregular variations. The logistic model applied was preferred and then used to forecast rainfall for the next 2 years (Alawaye and Alao 2017).

Yearly rainfall pattern was studied in Port Harcourt, South-Southern Nigeria by Amaefula (2018) and ARMA(1, 2) model was found most appropriate. Nyatuame and Agodzo (2018) in their study fitted ARIMA (3, 0, 3) and (3, 1, 3) models for annual rainfall pattern in Kpetoe and Tordzie regions in Ghana.

In Imo state, Nigeria, nine different SARIMA models were identified for monthly rainfall and compared based on AIC, SARIMA(0,0,0)x$(1,1,1)_{12}$ was preferred for predicting monthly rainfall in the state (Amaefula, 2019). ASARIMA$(P,D,Q)_s$ was introduced by (Amaefula, 2021) and compared with SARIMA(p, d, q)×$(P, D, Q)_s$ models, using Enugu monthly rainfall (EMR) as a case study, AIC showed that ASARIMA$(2, 1, 1)_{12}$ was preferred to all SARIMA(p, d, q)x$(P, D, Q)_{12}$ models that were identified.

As the population in Benin city grows due to mass movement of rural youths to urban areas, in search of livelihood, the demand for water becomes increasingly necessary and challenging not only for domestic and agricultural use but also for industrial use. Hence, the need for proactive research in modeling and forecasting of monthly rainfall pattern in Benin City, south-southern Nigeria.

The remaining part of the paper is organized as follows; section 2 deals with the material and methods, section 3 presents the data analysis and discussion of results and secction4 presents the conclusion

2. MATERIALS AND METHODS

The section deals with the materials and methods used in the study, such as sources of data collection, variable measurement, model specification framework, model identification, test for order of integration, model selection criteria and method of estimation.

2.1 Method and Sources of Data Collection

The rainfall data is obtained from published statistical bulletin by central bank of Nigeria (CBN, 2020) and in collaboration with Nigeria Metrological Agency(NMA). The monthly rainfall data covered the period of 1981M1-2016MM12 consisting of 432 observations.

2.2 Variable Measurement

The instrument for measuring rainfall is known as a rain gauge. It is a special kind of drum used to record the depth of rainfall collected and it is measured in millimeter.

2.3 SARIMA model Specification

Condition where univariate time series $\{X_t\}$ exhibits non-stationary characteristics as a result of either outliers, random walk, drift, trend, or changing variance, it is conservative to take first or second difference (d) to achieve stationarity. Hence, $\{X_t\}$ follows an autoregressive integrated moving average ARIMA(p, d, q) model of orders p, d and q of the form.

$$A(L)\nabla^d X_t = B(L)\varepsilon_t \tag{1}$$

where $\{X_t\}$exhibits seasonal pattern that is non-stationary, which may likely be observable via correlogram. Box and Jenkins(1976) suggested that $SARIMA(p,d,q)\times(P,D,Q)_S$ is given as

$$A(L)\Phi\left(L^S\right)\nabla^d \nabla_S^D X_t = B(L)\Theta\left(L^S\right)\varepsilon_t \tag{2}$$

where $\Phi\left(L^s\right)$ denotes lagged seasonal AR operator of order P and $\Theta\left(L^s\right)$ denotes lagged seasonal MA operators of order Q. The $\nabla^d = 1 - L$ is a regular differencing operator with $d \le 2$ and $\nabla_s^D = 1 - L^s$ is seasonal differencing operator and s is the seasonal order.

2.4 Adjusted SARIMA (ASARIMA) Model

According to Amaefula (2021), conditions where a regular time series data is integrated order zero I(0), but exhibits seasonal non-stationary behaviors, ASARIMA (P, D, Q)$_s$ can be more appropriate. And the model is of the form;

$$\Phi(\mathrm{L_S})\nabla_s X_t = \Theta(L_s)\varepsilon_t \tag{3}$$

Note that $\Phi(L_s)$ represents the seasonal autoregressive (SAR) operator and it is given as $\Phi(L_s) = 1 - \phi_1 L_{s\times 1} - \cdots - \phi_P L_{s\times P}$ and $\Theta(L_s)$ represents the seasonal moving average (SMA) operator, and it's given as $\Theta(L_s) = 1 - \theta_1 L_{s\times 1} - \cdots - \theta_Q L_{s\times Q}$.

Generally, ASARIMA(P,D,Q)$_s$ model with the inbuilt constant term is of the form;

$$\nabla_s X_t = \omega + \phi_1 \nabla_s X_{t-(s\times 1)} + \cdots + \phi_P \nabla_s X_{t-(s\times P)} + \theta_1 \varepsilon_{t-(s\times 1)} + \cdots + \theta_Q \varepsilon_{t-(s\times Q)} \tag{4}$$

where ω is the constant parameter and s is the seasonality index.

2.5 Model Identification

The autocorrelation function (ACF) and partial autocorrelation function (PACF) are useful in identifying both ARIMA and SARIMA models. For an AR(p) model, the ACF tails off slowly while for MA(q) model, the ACF cuts off after lag q. But the PACF of an MA(q) model tails off slowly whereas that of AR(p) model cuts off after lag p. The AR and MA models are known to show some duality relationships. In model building, it is prudence to prefer the use of mixed ARMA fit to either the pure AR or the pure MA fit.

2.6 Test for order of integration

The order of integration test (OIT) is another way of carrying out a unit root test and the reliability and aptness of OIT developed by Amaefula (2021) gives it leverage over conventional methods of unit root test. And it is of the form

$$z_t = \beta_0 + \phi T + \beta_1 z_{t-1} + \beta_2 z_{t-2} + \beta_3 z_{t-3} + e_t \tag{5}$$

where ϕ is the coefficient of the trend parameter T , β_0 is the constant term and it is optional to include it. The e_t is the random error term. β_1 , β_2 and β_3 are the autoregression coefficients. The parametric boundary conditions for integrated order one I(1) are; $|\beta_1| \geq 1, |\beta_2| < 1$, $|\beta_3| < 1$ and $\frac{|\beta_1|}{|\beta_2|} > 1$ and for I(2) it is expected that $|\beta_1| > 1$, $|\beta_2| \geq 1, \frac{|\beta_1|}{|\beta_2|} > 1$ and $\frac{|\beta_2|}{|\beta_3|} \geq 1$. For I(1), test the null hypothesis $H_{01}: \beta_1 < 1$ versus $H_{11}: \beta_1 \geq 1$ and for I(2) the null hypothesis $H_{02}: \beta_2 < 1$ versus $H_{12}: \beta_2 \geq 1$.

2.7 Criterion for Model Comparison

In model building, there are different information criteria for choosing the best fitted model. Conventionally, the Akaike information criterion (AIC) , Schwarz Bayesian information criterion(SBIC) and Hannan and Quinn (HQ) information criterion were introduced by *Akaike* (1974), Schwarz (1978), and Hannan and Quinn (1979) respectively, are most common. Another information criterion is that of final prediction error (FPE) which is attributed to Akaike (1969). If n is the sample size and RSS is the residual sum of squares, then, AIC, SBIC and HQ are given as follows.

$$\mathrm{AIC} = \mathrm{n} \times \ln\left(\frac{\mathrm{RSS}}{\mathrm{n}}\right) + 2 \times p \tag{6}$$

$$\mathrm{SBIC} = \ln\left(\frac{\mathrm{RSS}}{\mathrm{n}}\right) + \mathrm{p} \times \left(\frac{\ln \mathrm{n}}{\mathrm{n}}\right) \tag{7}$$

$$\mathrm{HQ} = \mathrm{n} \times \ln\left(\frac{\mathrm{RSS}}{\mathrm{n}}\right) + 2 \times \mathrm{p} \times \left(\frac{\ln(\ln \mathrm{n})}{\mathrm{n}}\right) \tag{8}$$

$$\mathrm{FPE} = \mathrm{n} \times \ln\left(\frac{\mathrm{RSS}}{\mathrm{n}}\right) \times \left(\frac{\mathrm{n}+\mathrm{p}}{\mathrm{n}-\mathrm{p}}\right) \tag{9}$$

In (6) - (9), *n* denotes the sample size, *p* represents the number of parameters estimated in the model, and RSS represents residual sum of squares. AIC tends to penalize models with larger number of variables. The difference between AIC and SBIC is in the severity of penalty for p. The penalty is far more severe in SBIC when $n > 8$. This tends to control the over-fitting tendency of AIC.

We will adopt the Amaefula forecast criteria (AFC) attributed to the work of Amaefula (2022) to identify the optimal model among the ones chosen by AIC, SBIC, HQ and FPE. The AFC is an extraction from the sum of squares deviation forecast criterion (SSDFC) introduced by Amaefula (2011, 2020). It is used as the optimal forecast performance identification test and it is of the form;

$$AFC = \left(G^T K G\right)^{\frac{1}{2}} \tag{10}$$

where $G = X_{t(l,i)} - \hat{X}_{t(l,i)}$, l = lead time, K = number of forecast values and should be reasonably large $X_{t(l,i)}$ = actual values of the variable corresponding to the i^{th} position of the forecast values, $\hat{X}_{t,(l,i)}$ = generated forecasts corresponding to the ith position of the actual values. The model with the lowest value of AFC is the optimal model.

2.8 Estimation Method

The identified ASARIMA model is estimated via iterative algorithm that calculates least squares estimates. The back forecasts at each iteration is computed and sum of squares error is calculated. For more details, see Box and Jenkins (1979).

3. RESULTS AND DISCUSSION

The results of the data analysis, graphical analysis, order of integration test, ACF and PACF for model identification, model comparison using information criteria, model estimation and diagnostic test summarily presented under this section.

3.1 Graph of Benin monthly rainfall (EMRF)

The time plot of Benin monthly rainfall (EMRF) is presented in Figure 1 below;

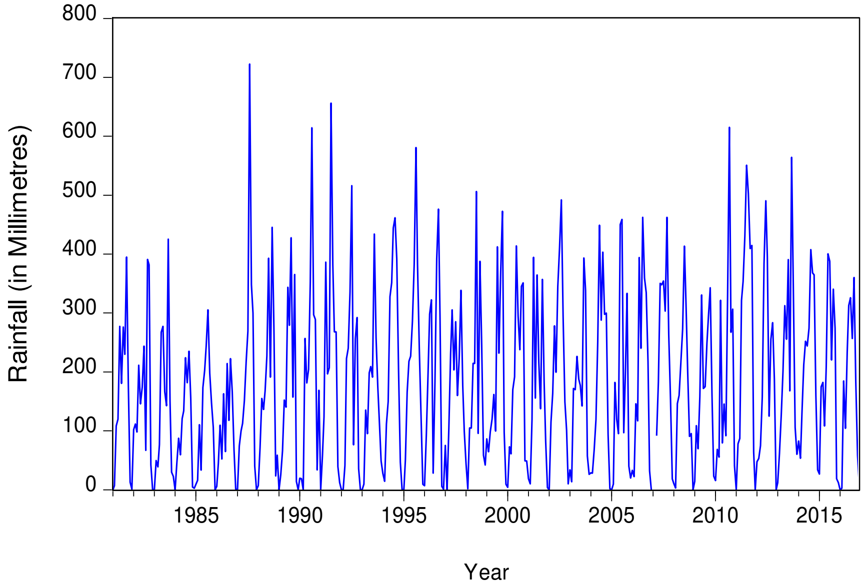

Figure1. Time plot of BMRF

The plot of BMRF monthly rainfall in Figure1 shows signs of seasonal non-stationary behaviours. The plot also exhibits non-trending pattern with the highest precipitation of 722.50 Millimetres in August, 1987 and lowest precipitation of 0.2 Millimeters in February,1992.

3.2 Order of Integration Test: Unit Root test

The unit root test result for the BMRF using AAR(3) OIT are presented in (11) and (12) below;

$$\begin{array}{llllll} \mathrm{BMRF_t} = 89.3974 + 0.0799t + 0.5557\mathrm{BMRF}_{t-1} + 0.0665\mathrm{BMRF}_{t-2} - 0.2086\mathrm{BMRF}_{t-3} + e_t \\ \text{prob.} \quad (0.0000) \quad (0.0958) \quad (0.0000) \quad (0.2262) \quad (0.0000) \end{array} \tag{11}$$

The result in (11) implies that BMRF is I(0) since $|\beta_1| = 0.5557 < 1,\ |\beta_2| = 0.0665 < 1,$ $|\beta_3| = |-0.2086| < 1$ and $\frac{|\beta_1|}{|\beta_2|} = \frac{|0.5557|}{|0.0665|} = 8.3564 > 1$ This result reveals that there is no unit root present in the variable BMRF. Therefore, it is sufficient to conclude that BMRF is stationary or I(0). Since the time plot in Figure1 has no evidence of trend, AA(3) OIT is estimated excluding the trend parameter as follows;

$$\begin{array}{l} \mathrm{BMRF_t} = 103.9141 + 0.5612\mathrm{BMRF}_{t-1} + 0.0695\mathrm{BMRF}_{t-2} - 0.2018\mathrm{BMRF}_{t-3} + e_t \\ \text{prob.} \quad (0.0000) \quad (0.0000) \quad (0.2072) \quad (0.0000) \end{array} \tag{12}$$

The AAR(3) OIT result in (11) yields the same conclusion as that of (12) since $|\beta_1| = 0.5612 < 1,\ |\beta_2| = 0.0695 < 1, |\beta_3| = |-0.2018| < 1$ and $\frac{|\beta_1|}{|\beta_2|} = \frac{|0.5612|}{|0.0695|} = 8.0748 > 1$ This result reveals that there is no unit root present in the variable BMRF. Therefore, it is sufficient to conclude that BMRF is stationary or I(0) in its regular series.

3.3 The Plot of ACF and PACF

The plots of autocorrelation function (ACF) and the partial autocorrelation function (PACF) for model identification are as presented in Figure2 and Figure3 below;

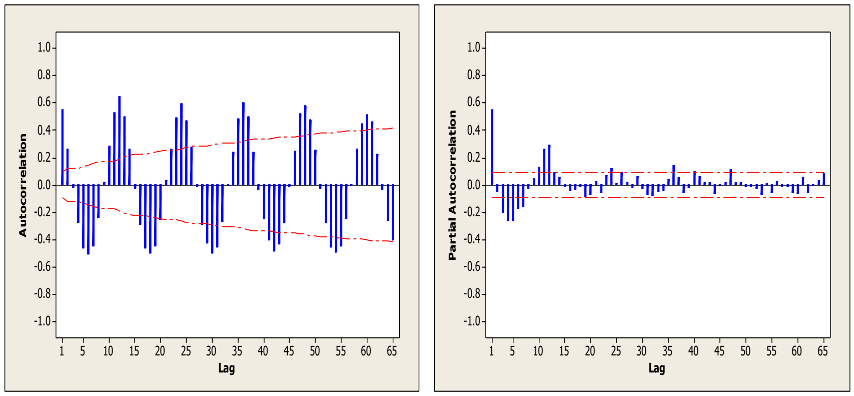

Figure2 Plot of ACF and PACF for BMRF

There is evidence of a varying seasonal pattern over time in the BMRF data as shown in the correlogram of Figure2 above. The seasonal non-stationary nature of BMRF requires seasonal differencing such as $\nabla^{12}BMRF_t = (1-L^{12})BMRF_t = BMRF_t - BMRF_{t-12}$ to make it seasonally stationary. The evidence of significant seasonal oscillation in the PACF up to the 15th lag is an indication of seasonal non-stationary nature of the BMRF data. Hence, seasonally differencing is key to achieving stationarity as aforementioned.

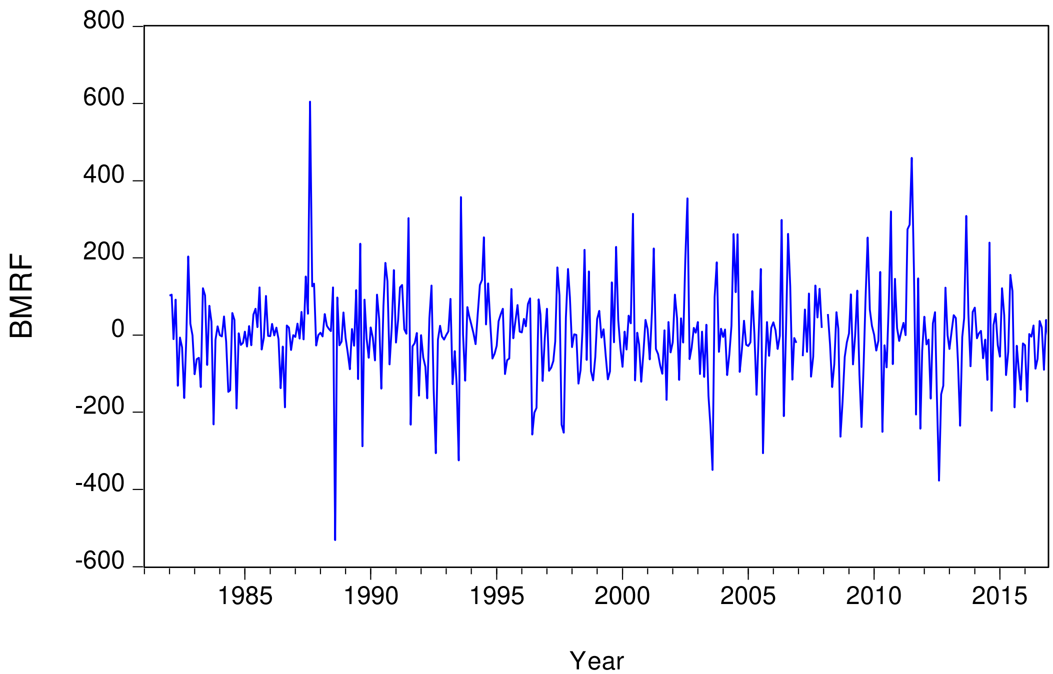

Figure 3. Time plot of $\nabla^{12}BMRF_t$ (1981M1 – 2016M12)

The time plot of 12 months seasonal difference for BMRF in Figure3 indicates that BMRF data points are concentrated around zero, depicting stationary condition.

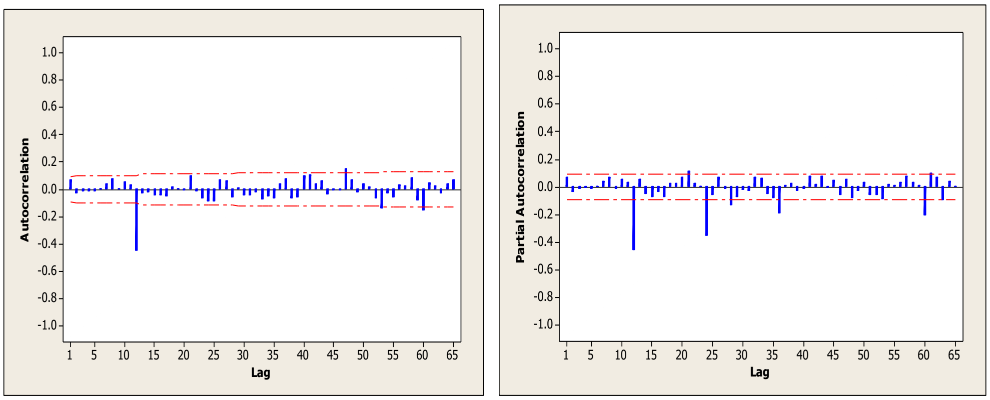

Figure4. Plot of ACF and PACF for $\nabla^{12} BMRF_t$ (1981M1 – 2016M12)

The ACF for $\nabla^{12} BMRF_t$ as shown in Figure4 reveals a significant spike at the 12th lag. And the PACF also shows significant spikes at the 12th, 24th 36th and the 60th lags respectively. However, there is no evidence of sinusoidal dying off slowly, and no evidence of either a pure AR process or MA process. Amaefula (2021) proposed $ASARIMA(P,D,Q)_{12}$ to be fitted to such time series with such characteristics. He suggested that fitting SARIMA(p, d, q) $\times$ (P,D,Q)$_{12}$ to such data generating process is not parsimonious and tantamount to estimating redundant parameters. Hence, fitting $ASARIMA(P,D,Q)_{12}$ is more appropriate.

Table 1. Model Selection using Model Information Criteria

S/No.	Model	AIC	SBIC	HQ	FPE	RSS
1	ASARIMA(1, 1, 0)$_{12}$	4037.78	4045.92	4016.79	4071.38	4906402
2	ASARIMA(2, 1, 0)$_{12}$	3978.80	3991.01	3947.34	4028.50	4261049
3	ASARIMA(3, 1, 0)$_{12}$	3953.49	3969.77	3938.81	4019.43	4000570
4	ASARIMA(4, 1, 0)$_{12}$	3952.97	3973.31	3900.55	4035.55	3977670
5	ASARIMA(5, 1, 0)$_{12}$	3926.40	3950.81	3863.52	4024.98	3723750
6	ASARIMA(0, 1, 1)$_{12}$	3879.52	3887.66*	3858.57	3911.68*	3401715
7	ASARIMA(0, 1, 2)$_{12}$	3879.27	3891.48	3847.85	3927.61	3384520
8	ASARIMA(0, 1, 3)$_{12}$	3883.66	3899.93	3841.76	3948.33	3403679
9	ASARIMA(0, 1, 4)$_{12}$	3885.85	3906.19	3833.47	3966.90	3405612
10	ASARIMA(0, 1, 5)$_{12}$	3888.43	3912.84	3825.58	3985.97	3410641
11	ASARIMA(1, 1, 1)$_{12}$	3880.67	3892.87	3849.24	3929.02	3395431
12	ASARIMA(1, 1, 2)$_{12}$	3875.17*	3891.44	3833.27	3939.68	3337485
13	ASARIMA(1, 1, 3)$_{12}$	3882.79	3903.13	3830.41	3963.77	3381602
14	ASARIMA(1, 1, 4)$_{12}$	3884.21	3908.62	3821.88	3981.63	3377494
15	ASARIMA(1, 1, 5)$_{12}$	3884.74	3913.22	3813.35*	3998.65	3366459
16	ASARIMA(2, 1, 1)$_{12}$	3879.59	3895.87	3837.69	3944.18	3371794
17	ASARIMA(3, 1, 1)$_{12}$	3884.98	3905.32	3832.60	3966.01	3398771
18	ASARIMA(4, 1, 1)$_{12}$	3956.05	3980.46	3893.14	4055.43	3988039
19	ASARIMA(5, 1, 1)$_{12}$	3952.57	3981.05	3879.19	4068.65	3938276

The symbols(*) indicates the chosen model specification by AIC, SBIC, HQ and FPE respectively.

The result of model selection in Table 1 indicates that ASARIMA (1, 1, 2)$_{12}$, ASARIMA (0, 1, 1)$_{12}$ and ASARIMA (1, 1, 5)$_{12}$ are chosen by AIC, SBIC and HQ respectively to other ASARIMA specifications. The FPE choice agrees with that of SBIC. The smallest value of RSS aligned with AIC choice, however, the two selected models are then subjected to forecast performance test as presented in Table2 below;

Table 2. Optimal forecast performance identification test using AFC

S/No.	Information Criteria	Model	AFC
1	AIC	ASARIMA(1, 1, 2)$_{12}$	96.7239*
2	SBIC and FBE	ASARIMA(0, 1, 1)$_{12}$	99.1639
3	HQ	ASARIMA(1, 1, 5)$_{12}$	101.245

The symbols(*)indicates the best model.

The result in Table2 above shows that optimality identification aligns with the choice of AIC and HQ, which indicates that ASARIMA (1, 1, 2)$_{12}$ yields the optimal forecast values and it's considered the best model. Therefore, the estimate of the model is presented in Table3 below.

Table3. Final Estimates of ASARIMA(1, 1, 2)$_{12}$ Parameters

Type	Coef	SE Coef	T	P
SAR 12	-0.5098	0.5910	- 0.86	0.389
SMA12	0.4165	0.5698	0.73	0.465
SMA24	0.5501	0.5530	0.99	0.320
Constant	2.5864	0.3477	7.44	0.000
Differencing: 0 regular, 1 seasonal of order 12				
Number of observations: Original series 432, after differencing 420				
Residuals: SS = 3337485 (backforecasts excluded)				
MS = 8023 DF = 416				

The estimated model in Table3 produces the minimum residual sum of squares, and the model form is presented in (13) below.

$$\nabla_{12}X_t = 2.5864 - 0.5098\nabla_{12}X_{t-12} + 0.4165\varepsilon_{t-12} + 0.5501\varepsilon_{t-24} \quad (13)$$

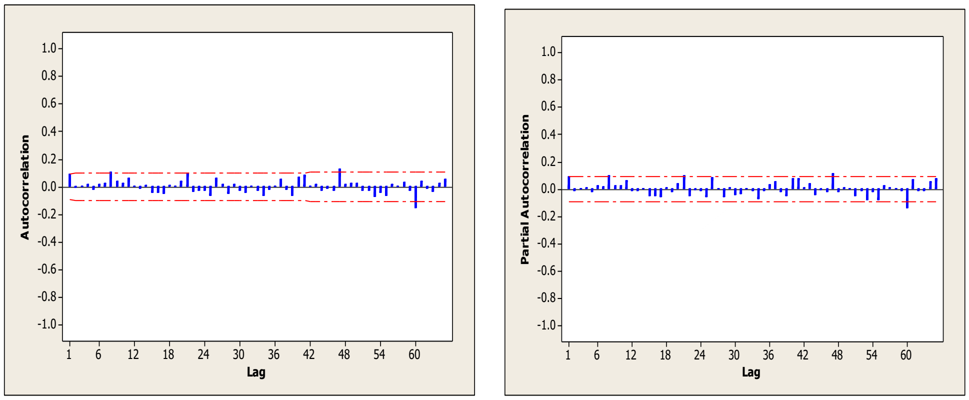

Figure5. ACF and PACF of Residuals for BMRF

The absence of no significant spikes up to 48th lag in the autocorrelation function (ACF) and partial autocorrelation function (PACF) of the model residuals as shown in Figure5 strongly indicate that the residuals are uncorrelated and the fitted ASARIMA$(1, 1, 2)_{12}$ model for BMRF is adequate and suitable for generating forecasts.

Table4. Modified Box-Pierce (Ljung-Box) Chi-Square statistic

Lag	12	24	36	48
Chi-Square	11.9	20.7	29.3	46.7
DF	8	20	32	44
P-Value	0.154	0.415	0.606	0.363

The result of Modified Box-Pierce Chi-Square statistic in Table4 shows that the residuals of the fitted model are not correlated up to 48th lag as the p-values are not significant. Therefore, the fitted ASARIMA $(0, 1, 1)_{12}$ model is adequate.

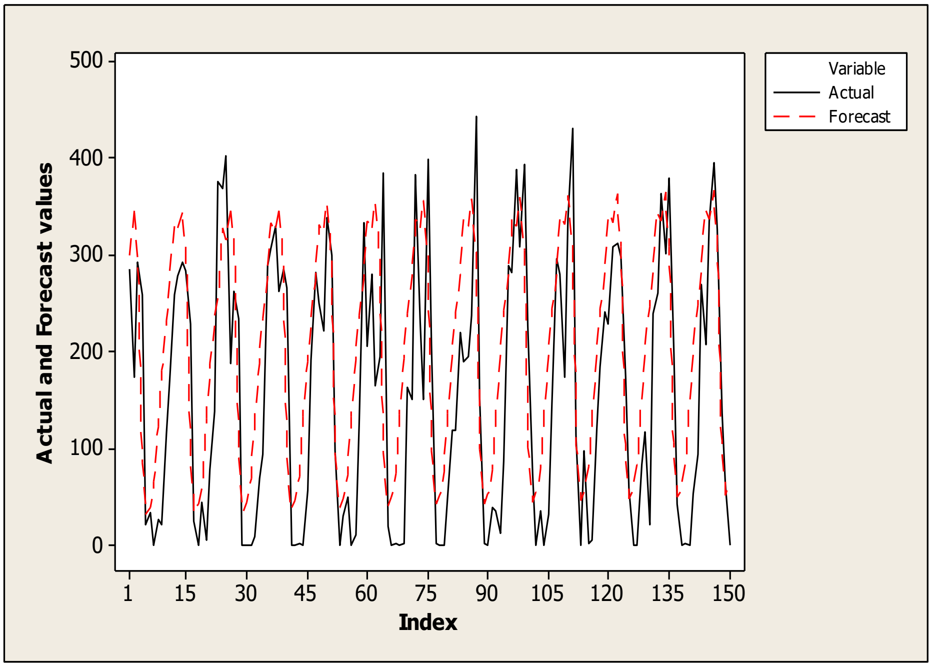

Figure 6. Plot of forecast values using fitted ASARIMA$(1, 1, 2)_{12}$ model for BMRF model (*lead time = 150, origin =283*)

The generated values with the fitted ASARIMA $(1, 1, 2)_{12}$ model are much related to the actual values.

3.4 DISCUSSION OF RESULTS

When a time series data is stationary at its regular level but exhibits some non-stationary seasonal behaviour, Amaefula(2021) showed that, it is better to model such seasonal data using ASARIMA$(P, 1, Q)_s$ than seasonal ARIMA models. Again, ASARIMA $(P, 1, Q)_s$ model eliminates parameter redundancy because it is more economical and parsimonious than SARIMA Models. The BMRF data used in this paper is found to be stationary and significant at 1% level. The ACF in Figure2 reveals that BMRF has seasonal characteristics and the PACF shows some seasonal fluctuations, indicating the need for seasonal differencing in the model.

Nineteen possible ASARIMA$(P, 1, Q)_{12}$ models were compared using AIC, SBIC, HQ and FPE, respectively. ASARIMA $(1, 1, 2)_{12}$, ASARIMA $(0, 1, 1)_{12}$ and ASARIMA $(1, 1, 5)_{12}$ were selected having found to have the smallest values for AIC and SBIC and HQ, respectively. FPE aligns with SBIC. The three models were further subjected to optimality identification using AFC and ASARIMA $(1, 1, 2)_{12}$ is found to be the optimal model, yielding the most precise forecasts. The diagnostic test shows that the fitted model is adequate. And the generated forecasts are quite close to the actual values.

4. CONCLUSION

The findings of the study reveal that model identification using a particular information criterion does not guarantee optimal identification and model out-put performance. A comparison of the preferred models by different information criteria using AFC is recommendable as empirically demonstrated in this study. However, ASARIMA$(1, 1, 2)_{12}$ model is found to be optimal and adequate in predicting BMRF. Therefore, ASARIMA$(1, 1, 2)_{12}$ is strongly recommended for predicting monthly rainfall and its effects in Benin City South-Southern Nigeria.

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