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USE OF GEOMETRIC SEQUENCE IN PRODUCTION CONTROL AND MACHINE LIFE MAINTENANCE

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ABSTRACT

The usefulness of machine in industries and other areas of life cannot be overemphasized. This work considers the effect of wears and tears of machines as a result of usage and the value of depreciation with their effect on productivity on the long run of an industrial machine. A consideration of machine whose life depends on the rate of depreciation with time and magnitude of usage in production process has been identified to exhibit some functional expression. The life span of a machine with effect from the date of acquisition in conjunction with its capacity at book value is modeled as a geometric sequence a, ar, ar^2,…, ar^{n-1},... . This model is used to demonstrate some managerial decisions to maintain optimal production from machine usage with less break down time. The practical application of these expressions is illustrated and the method proves effective in machine life management. The method also proves helpful in production control for optimal yield for the industry.

Keywords: geometric sequence, production control, machine life management

INTRODUCTION

Machines play vital role in production of goods and services in industries and other establishments. Science play very important role in machine manufacture and mathematics as a science is useful in the development of models to maintain optimum production process and usage. Machine usage and system processes can be modeled in such a way that cost of maintenance is minimized while production is optimized (Ball et al 1998).

The life span of a machine is a function of the magnitude of usage and maintenance. The book value of the machine depreciates as the time of usage increases. The capacity of the machine is also taken into consideration as usage above capacity affects productivity and life span (Yeh, 2009). The geometric sequence is a sequence of the form a, ar, ar^2,…, ar^{n-1},..., where "a" is the first term, "r" is the common ratio, "n" the number of terms (years) and "ar^{n-1}" is the last term (Zheng 2003). The life span of a machine with effect from the date of acquisition together with it's capacity at book value is modeled as a geometric sequence.

Emphasis is on the need for modern technology to be applied to the production of physical objects and also apply managerial skills to reduce wastages and losses in the process while maximizing productivity simultaneously. This is done by applying sequence of processes/rules on the organization and maintenance of equipments by management.

When a machine is producing above its normal capacity at any time, it undergoes stress, on the other hand if under relax condition it produces below normal capacity. In either of these two conditions the machine is affected positively or negatively. The effect on its life span and overall production during its economic life under stress is drastically reduced in productivity but prolonged life under relax situation. The annual adjustment of the production rate reflects the normal production capacity for the year to ensure either stress or relax.

The maintenance of machine is a significant part of the production management system. Machines not being maintained lead to frequent breakdown and consequently early death. Knowing that a machine gradually looses value through usage calls for the management to provide for depreciation and ensure proper and efficient usage. Proper maintenance culture is also required to uphold the production line and ensure minimal expenses. Machine facilitates production and enhances profit maximization through high yield in productivity (Zao, Yue and Tien 2010). Due to usage machine life span is often affected because of wear and tear and breakdowns. Consequently the life span of a machine is limited and can even be shortened if not properly and timely checked and managed (Zao and Tien (2010)). The rate at which machine is used affects its productivity.

Machines are more efficient at their newest age than towards their expiration. The period within which it is in active production and profitable use (its economic life) is of great concern to the industrialist (Rowell, Pope and Sherman 1992). The length of time during which the machine is profitable is of considerable importance to the management and efforts are made to sustain it (Gao 2014). When a machine is no longer being profitable it loses its value and the usage may contribute negatively to the system (De Garmo and Carnada 1973). At the expiration of its economic life the industrialist may not wish to possess it because of the reduced value. Hopeman 1971 posits that depreciation simply means the decrease in value of physical properties with the passage of time. However skillful planning and inventory control can provide smooth production operations which will utilize the equipment to near capacity. If they are not used at their near capacity they will still continue to depreciate as time goes by, but there will be a contribution to overhead from profit which would result if production were maintained at or near capacity. The actual production rate at or near capacity is desirable and can be derived using geometric sequence $S_n = ar^{n-1}$ where "a" is the initial value, "r" is the common ratio observed among the terms while "n" is the terms of occurrence.

Machine depreciates both in monetary value and in production capacity. These are evidenced in the number of breakdown periods and its book value at any period of its life due to depreciation. Machine life experiences certain effect due to stress as it produces above its normal production capacity (Sheu 1999). For instance if a machine is producing continually at its initial production capacity it will be out of use earlier than expected even though its total production remains high. Machine contributes more to the overhead profit when it is producing at or near capacity (Hopeman 1971). When machine has entered its stress condition, it no longer contributes to the overhead profit although production at salvage cost ensures longer life span for the machine but production will not be maximized (Boshuizen 1994) . The only maximization rate of production to contribute to the overhead profit is the value of the production rate calculated at the salvage year (or cost). Chase 1989 supports that the life of a machine is not estimated in years but rather in terms of total number of operations it may be reasonably expected to perform before wearing out. The estimation of number of years the machine would serve is based on the fact that it operates at some average hour per day. This average hour production rate can be used to determine its economic life.

When machine produces above its normal capacity it suffers stress effect and after its salvage year it's stress value will be greater than the normal capacity. At this instance the machine no longer contributes to the overhead profit (Florinni et al 1998). The stress and relax values are simply the difference between the actual production rate and the normal capacity for the year. According to Lam 1991, when this difference is greater than the normal capacity it is not advantageous for the industry. The stress effect actually indicates negative value so that the stress effect (E_s) is less than zero.

MODEL FORMULATION

The life span of machine from the date of acquisition along with its capacity at book value is modeled as a geometric sequence as follows;

Assumptions:

The rate of depreciation in any one year to the book value at the beginning of that year is assumed constant throughout the life of the machine and is denoted by "k".

The value of "k" is derived intuitively as follows:

Let "X" be the cost, "k" the rate of depreciation and "d" depreciation of the machine.

The depreciation for the first year is d_1 = kX_n where n = 0 i.e. d_1 = kX_0

Depreciation during the second year is d_2 = kX_{n-1} = kX_1

Depreciation during the third year is d_3 = kX_2

Hence depreciation for the nth year is d_n = kX_{n-1}

The savage value at the L_{th} year is L_n = X|1-k|L

Thus the machine depreciates at the rate of k% of its value at the beginning of the year and the exact value at the end of the year is

L_1 = |1-k|X (1)

where X is the book value at the beginning of the year.

Similarly the book value at the end of the second year is (1-k)(1-k)X =(1-k)^{2}X; and

the third year would then be (1-k)(1-k)(1-k)X = $(1-k)^3X$ ……. i.e. a geometric sequence.

Thus the entry value of each year follows a geometric progression

$L_n = (1-k)^{n-1}$X, n = 1, 2, 3, … (2)

Therefore the machine depreciates if its normal capacity has reduced so that production fixed at higher capacity than normal affects its life. Hopeman 1971 therefore insists that machine can quickly determine difference in dimensions, pressure and temperature and can respond quickly to deviation from the norm. in other words machine can either be producing at its maximum capacity at that age or not producing at all. It may be over producing under stress or under-producing at a relax condition. The need to balance production line is inevitable and this arises not only to meet the demand but also to preserve the machine life. The aim should be to operate the machine to ensure a maintenance free performance. This is done considering the initial production capacity that depreciates at k%.

The capacity at the beginning of Nth year of production becomes;

C = XR^{N-1} (3)

where C is capacity, R =1-k and N is age at salvage value.

Gao (2014) stated that the depreciation is calculated in monetary values of the machine based on its book value at any age. The purpose is to assess the machine's profitable age at which loss in production will be minimal. The monetary value at which the machine can be disposed off is referred to as the salvage value. At the age where the machine is considered for disposal is a crucial point in decision making in industrial activities. This age can be related to depreciating rate k% at salvage value and expressed by;

S = AR^{N-1} (4)

Where S = monetary value at salvage year, A = initial book value, N = age at salvage value, K = rate of depreciation and R =1-k.

Now S and A are constants and R^{N-1} is a function of n.

Differentiating (iv) with respect to N, gives

$$\frac{dS}{dN} = \frac{dS}{dn}\frac{dn}{dN}\ |R^{N-1}|$$

$$= \frac{A}{R}\frac{d}{dN}|R^N|$$

R is a constant hence we differentiate logarithmically to get;

$$0 = \frac{A}{R}\text{LogR}$$

$$= AR^{N-1}\text{Log R}, \quad \text{Log R} \neq 0$$

Set AR^{N-1} = 0 to get

$$N = 1 - \frac{Log\ A}{LogR} \quad (5)$$

Equation (5) expresses the maximum life span of the machine at k% depreciation.

Now considering production control in the industry, the production rate (p) under a total stress is fixed at the initial production capacity.

Suppose the rate of change in production value as a result of depreciation is such that $p_i > p_{i-1}$,

i = 1,2,3,…. such that

$\frac{P_1}{P_2} = \frac{P_2}{P_3}$ is a constant (Q)

Then P_1-P_2 = Q (P_2- P_3) and P_2 – p_3) = P_1 – QP_3.

$$Q- 1 = (\frac{P_1 - Q\ P_3}{P_2}) \quad (6)$$

Now fix the value of P_i so that $P_1 = P_2 = P_3$ = P where P is the fixed rate of production under stress.

It follows from (vi) that

$$Q -1 = (\frac{p-pQ}{p})$$

And Q-1 = 1-Q which implies that Q = 1.

Since we are dealing with stress condition, we take Q = -1

We now state that;

$$\frac{CR^{N-1}-p}{CR^{N-1}} = -1 \quad (7)$$

Where C = initial capacity, P = production rate, R = 1-k (a function of depreciation and N = age of the machine under consideration..

Equation (7) is used to determine the economic life of the machine at the production rate.

On the other consideration of relax condition, the salvage production capacity is the same as equation (7) except that Q = 1 so that

$$\frac{CR^{N-1} - P}{CR^{N-1}} = 1 \quad (8)$$

Incidentally production is not minimized by fixing P at initial production capacity or salvage production capacity. Rather when salvage year has been determined (say N) equations (7) or (8) is applied to determine the value of P which is at or near capacity of the machine at any given year or periodic time of change.

The application of geometric progression principles demonstrated in equations (1) – (4) can be used to maintain machine life to be in use for longer period. This will in addition bring high production with minimized machine breakdown time. The number of years of profitable production enables the industry to operate at the rate where machine can contribute to its profits rather than deficiency. Chase 1989 supports that the life of a machine is not estimated in years but rather in terms of total number of operations it may be reasonably be expected to perform before wearing out. The estimation of number of years the machine would serve is based on the fact that it operates at some average hour per day. This average hour production rate can be used to determine its economic life.

RESULTS

A printing machine is bought for 100,000 naira with initial capacity of printing three million papers a year at 6 hours daily operation. Its depreciation value is assessed at 5% per annum. The company decided to dispose off the machine whenever its book value has reduced to 40,000 naira. When would the machine be disposed off? At what rate of production will the machine provide maximal total production?

The solution is as follows;

Initial book value = #100,000, Depreciation rate = 5%, Normal capacity at any year(R) = 95%

Initial production capacity = 3,000,000 per annum, Salvage value = #40,000 and

Book value at Nth year = AR^{N-1}

Applying equation (iv), substitute the values and solve for N.

$40{,}000 = 100{,}000\ (0.95)^{N-1}$

$= 0.95^{N-1} = 0.04$

Taking log of both sides to solve for N gives 19 years.

Hence the machine will be disposed off at the end of the 19th year of 6 hour daily operation at 3,000,000 per annum.

Now using N= 19 and substitute in (vii) obtains P. i.e.

$$\therefore \qquad \frac{CR^{N-1} - P}{CR^{N-1}} = -1$$

Where C = initial production capacity, P = average production rate, R = 0.95, N = 19

Then $P = 2CR^{N-1}$

$= 2\ (3{,}000{,}000)\ (0.95)^{18} = m6{,}000{,}000\ (0.95)^{18}$

$= 2{,}383{,}286.$

The average rate of production to ensure that the machine operates for 19 years with less overhead cost is 2,383,286. This is the maximizing production rate (MPR) for the machine during the period.

With this value of P a total of 45,282,432 units would be provided. The appendix I and ii below show the various salvage values and the stress/relax effect of machines being to use. The values shown attest to the usefulness and better management of machines for optimal use.

CONCLUSION

The need to manage resources in an industry is a significant part of the management process. Management goals are geared toward profitability through minimizing cost and optimizing gain. Machines are very vital components of industries and must be operated at a level to achieve reasonable output. As a result of usage, machines depreciate in value and a continual usage may lead to low output if proper maintenance management is not adapted.

The application of geometric progression principles demonstrated in equations (1) – (4) can be used to maintain machine life to be in use for longer period. This will in addition bring high production with minimized machine breakdown time. The number of years of profitable production enables the industry to operate at the rate where machine can contribute to its profits rather than deficiency. Chase 1989 supports that the life of a machine is not estimated in years but rather in terms of total number of operations it may be reasonably expected to perform before wearing out. The estimation of number of years the machine would serve is based on the fact that it operates at some average hour per day. This average hour production rate can be used to determine its economic life.

It has been demonstrated here that the geometric sequence is a tool for industrial management on depreciable machines. The method has also demonstrated that the expected yield of machines can be fixed. It is also shown that the capacity of the machine decreases due to depreciation. Geometric sequence has been employed to obtain a model that minimizes cost of production and consequently maximizes profit by firms that employ the use of machines in their operations.

REFERENCES

Ball,G.,Stephenson,G,Smith,G.,Wood,L.,Coupland,M and Crawford,M (1998) Creating a diversity of mathematical expression for tertiary students. International *Journal of Mathematics Education, Science Technology, 29(6) 103-109.*

Boshuizen,F.A. (1994) A replacement model with general age-dependent failure rates.
JSTAT PLAN INFER.

Churchman, C.N., Ackoff, R.L. and Amoff, E.R.(1961) Introduction to Operations Research, 6th Edition. John Wiley and sons Inc. N.Y.

DeGarmo, E.P. and Canada, J.R.(1973) Engineering economy, N.Y. Macmillan Publishing company Inc.

Florinni,G.R., Macmillan,J. and Accost, A.P. (1998) Developing Activities for Mathematics and Computer Education. 12(2) Spring

Gao, Q.C.L.(2014) An optimum replacement policy for a degenerate system with two types of failure state. J COMPUTAPPL MATH.

Hopeman, R.J. (1971) Production concept Analysis Control,2nd Edition Charles Merill Publishing company, Columbus.

Lam, Y. (1991) An optimal reparable replacement model for deteriorating systems. *Journal of applied probability 28(4):58-63*

Rowell,J.A,Pope,S and Sherman,B.F.(1992) Relating theory to practice in Teacher Education. *Education for teaching. 18(2)*

Sheu, S.H. (1999) Extended optimal replacement model for deteriorating systems. *European Journal of Operations Research 98,62-69.*

Yeh, l. (2009) A geometric process – shock maintenance model. IEEE T RELIAB.

Zang, H.Y, Yuan,Z. and Yeh, L.(2002) A geometric process equivalent model for multi-state degenerative system. *European Journal of Operations Research 101,84-89.*

Zao,B.,Yue,D. and Tien,R.(2010) A geometric process maintenance model of replacement system with a replaceable repair facility. *Journal of mathematical Sciences 5(2): 58-61*

Zheng,Y.L. (2003) A geometric process maintenance model for a random environment. IEEE-T Reliab.